# Chapter One

# INTRODUCTION

## 1.1 Volatility Models

Volatility models are dynamic models that address unequal variances in financial time series, the first and formal volatility model is the Autoregressive Conditional Heteroskedasticity (ARCH) model by Engle Robert (1982). The history of ARCH is a very short one but its literature has grown in a spectacular fashion. Engle's Original ARCH model and it various generalisation have been applied to numerous economic and financial data series of many countries. The concept of ARCH might be only a decade old, but its roots go far into the past, possibly as far as Bachelier (1900), who was the first to conduct a rigorous study of the behaviour of speculative prices. There was then a period of long silence. Mandelbrot (1963) revived the interest in the time series properties of asset prices with his theory that random variables with an infinites population variance are indispensable for a workable description of price changes. His observations, such as unconditional distributions have thick tails, variance change over time and large (small) changes tend to be follow by large (small) changes of either sign are stylised facts for many economic and financial variables.

Empirical evidence against the assumption of normality in error term of stock return has been ever since the foremost journals by Mandelbrot (1963). Clark (1973) debated that prices increase can be defined by a uniform Paretian distribution with an attribute exponent lower than two, hence an infinite variance along with exhibiting. Financial time series are usually characterise with clustered volatility, unequal variance and Leptokurtic

(Mandelbrot, 1963). Additional characteristic always come across is the alleged leverage impact (Black 1976), which is negative correlation between changes in volatility and stock prices. Such work is scared in Nigeria Stock Exchange Market and this type of observations have lead to applying a broad conditional volatility models to ascertain and forecast volatility in financial time series.

A fundamental principle of the least squares model presumes the predicted worth of all error has constant variance at any time, homoskedasticity assumption. Data through which the contrast of the error innovations are unequal, where upon the error innovations is possibly anticipated as being greater for sometimes to otherness, these is pronounce to come down with heteroskedasticity. A major habitual caution resulting from heteroskedasticity is that, confidence intervals evaluated by orthodox methods will be too restricted with standard errors, leading to a wrong sense of precision. Rather than debating this as a lacuna to be error-free, ARCH and GARCH models project heteroskedasticity as a variation to be model after. Prediction obtained for the variance of errors term and least squares deficiencies are corrected. The major interest is prediction, especially in economic and finance usage. The new evolution in determining standard errors, known as "robust standard errors," minimised

Worries about heteroskedasticity. If the hypothesis dimension is big, then robust standard errors give quite a proven valuation of standard errors even with heteroskedasticity. If the hypothesis dimension is little, then the heteroskedasticity necessity amendment that does not strain the coefficients, and only symptomless redress the standard errors, can be argued. Nevertheless, periodically logical question being asked the applied econometrician is the efficiency of the prognosis of the model. At this rate, the major problem is the variation of the error terms and what cause them to be big. This investigation always shows up in financial implementation where return on asset is the dependent variable and the risk level of returns represent the variance. These are time series applications, but it is nevertheless expected that heteroskedasticity is a problem. A superficial glance at financial information propose that some time periods dangerous than others; this means that, periodically the anticipated worth of the enormous error terms larger than others. Besides, these dangerous times are not distributed immethodically over quarterly or yearly data. Rather, there is a measure of autocorrelation in the dangers of financial yield. Financial analysts, viewing the plots of daily returns such as in Figure 1,displayed what is called volatility clustered that is volume of the returns differ with time, the ARCH and GARCH models are formulated for issues like this. They have turn to comprehensive instruments for tackling time series heteroskedastic models.

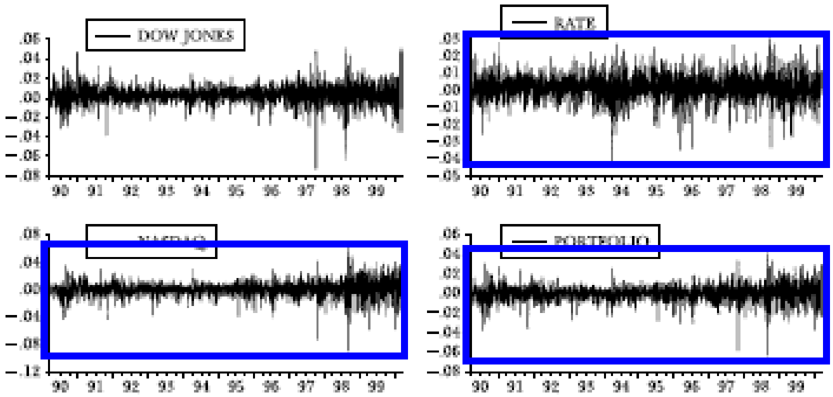


Figure 1.1: **Nasdaq, Dow Jones and Bond Returns between 1990 - 1999**

### 1..1.1 Stylised Facts About Volatility Models

Formulation of GARCH models was reported for practical balance in financial information, some of the characteristics they have in common are:

1. Presence of non-stationary in Asset price while returns are commonly stationary. fractionally integrated are found in some financial time series.

2. Little or no autocorrelation occurs in return series.

3. Their exist non- linear relationship between successive observations because squared of series with serial independence are rejected.

4. Returns are characterised by Clustered Volatility.

5. Thick-tailed distribution is favoured while normality is rejected.

6. Leverage effect occurs in some series, which happens when changes in stock prices is negatively correlated with changes in volatility.

7. Volatilities of large(small) securities regularly move together.

### 1.1.2 Justification For The Use Of Arch Models

Decomposition theorem opined by Wold's (1954) stated that any covariance stationarity $(y_t)$ may be written as one sided moving average representation, a linearly stochastic with a square- summable and the total of a linearity deterministic component

$$y_t = d_t + \mu_t \qquad 1.1$$

$\mu_t$ is a linearly regular covariance stationary stochastic process and $d_t$ is the linearly determistic given by

$$\mu_t = \beta(L)\varepsilon_t \qquad 1.2$$

Where,

$$\beta(L) = \sum_{i=0}^{\infty} b_i L_i, \qquad \sum_{i=0}^{\infty} b_i < \infty, \qquad b_0 = 1 \qquad 1.3$$

$$E\,[\varepsilon_t \varepsilon_\tau] = \begin{cases} \sigma_\varepsilon^2 \; ; & if \quad t = \tau \\ 0 \quad ; & otherwise \end{cases} \qquad 1.4$$

The uncorrelated error sequence need not to be normally and independently distributed. Independent errors are characteristic of conditional variance and non linear time series in common. Assuming that $y_t$ is transformed covariance variable from non-stationary to stationary linear innovation with identical independent variable . The unconditional mean and variance are invariant in time, respectively.

$$E(y_t) = 0 \qquad 1.5$$

$$E(y_t^2) = \sigma_\varepsilon^2 \sum_{i=0}^{\infty} b_i^2 \qquad 1.6$$

Given below is the conditional mean which is time varying:

$$E(y_t / I_{t-i}) = \sum_{i=1}^{\infty} b_i \varepsilon_{t-i} \qquad 1.7$$

$I_{\ t-i} = \left(\varepsilon_{t-1,}\ \ \varepsilon_{t-2\ ,}\varepsilon_{t-3,}\ \ldots \varepsilon_{t-i.}\right) \mathit{is\ the\ information\ set}$ and dynamics conditional variance cannot be captured adequately by the model. The conditional variance of $y_t$ is constant at

$$E[(y_t - E[y_t/I_{t-i}])^2/I_{t-i}] = \sigma_\varepsilon^2 \qquad 1.8$$

The conditional prediction at p - step ahead is

$$y_{t+p}/I_t = \sum_{i=0}^{\infty} b_{p+i}\varepsilon_{t-i}$$

The prediction error is

$$y_{t+p} - E\left[y_{t+p}/I_t\right] = \sum_{i=0}^{\infty} b_i\varepsilon_{t+p-1} \qquad 1.9$$

While the conditional prediction error variance is

$$E\left[\left(y_{t+p} - E\left[y_{t+p}/I_t\right]\right)^2/I_t\right] = \sigma_\varepsilon^2 \sum_{i=0}^{p-1} b_i^2 \qquad 1.10$$

If $p\ \rightarrow\ \infty$ in conditional prediction error variance then convergence to Unconditional variance $\sigma_\varepsilon^2 \sum_{i=0}^{\infty} b_i^2$ is reached. The conditional prediction error variance relies solely on t at any p and not $I_t$. The simple independent identical distributed innovations model cannot capture the information which is obtainable at time t.

Pioneering a model by way of a proposal, Engle (1982) established a formal conditional variance which is time varying; ARCH processes utilising lagged white noise. Proofs of his work predicated on scientific research proceedings showed that to project dynamic deportment of conditional variance, high order of ARCH(p) model is needed. The GARCH model proposed by Bollersllev (1986) efficaciously established this condition as it is premised on an illimitable ARCH designation which diminishes the number of calculated parameters from illimitability to two. Clustered volatility and Leptokurtosis can be captured by ARCH and GARCH but not adequately due to underlying symmetric error innovations. Other extensions of nonlinear of GARCH type have been suggested to address this challenge, such as the Asymmetric Power ARCH (APARCH) model by Ding (1993), the Exponential GARCH (EGARCH) model by Nelson (1991) and the so-called Glosten- Jagannathan- Runkle (GJR) model by Glosten (1993).

Fat tails property of high frequency of financial time series cannot be adequately capture when using GARCH (p,q) model. To overcome this lacuna, the Student's t- distribution was used to address the thick t

ails occurrence in financial time series model, Beine *et al;* (2002), likewise to encapsulate skewness, asymmetric stable density was used in place of normal density; Liu and Brorsen (1995). Skewed Student-t distribution was used to capture both Kurtosis and skewness by Fernandez and Steel (1998), while Lambert and Laurent (2001) incorporated skewed distributions into GARCH frameworks. Haris *et al;* (2004) capture leverage effect of daily returns and skewness with other extensions of non-linear GARCH models along with skewed generalised student-t distribution.

A comparative studies and forecast performance of conventional GARCH and other extensions of GARCH models such as asymmetric types have been extensively investigated by Pagan *et al*; (1990) which are in favour of asymmetric GARCH models., while Baillie *et al*; (1998) investigated the performance of non-normal and normal densities of GARCH models in terms of model selections and forecast performance considering minimal values of its model selection criteria and loss functions.

This project investigates the suitable models and forecasting performance of volatility models assuming their error innovations to be Student- t , Normal, Generalised Error distribution, Length Biased Scaled –t distribution and Generalised Beta Skew t-distribution. A comparison studied was carried out between symmetric and asymmetric innovations of GARCH (p, q) and APARCH (p, q) models.

### 1.1.3 NOTATION

1. ARCH : Autoregressive Conditional Heteroskedasticity
2. GARCH: Generalised Autoregressive Conditional Heteroskedasticity
3. APARCH: Asymmetric Power Autoregressive Conditional Heteroskedasticity
4. GED : Generalised Error Distribution
5. STD: Student-t Distribution
6. GBST: Generalised Beta Skewed Student-t
7. GLBST : Generalised Length Biased Student-t

14. AR: Autoregressive
15. BIC: Bayesian Information Criterion
16. AIC: Akaike Information Criterion
17. HQIC: Hann Quinn Information Criterion
18. SBIC: Schwertz Bayesian Information Criterion

| | | |
|---|---|---|
| 19. | RMSE: | Root Mean Square Error |
| 20. | AMAPE: | Adjusted Mean Average Percentage Error |
| 21. | P VLAUE: | Power Value |
| 22 | T- RATIO: | Student-t Ratio |
| 23. | SD: | Standard Deviation |
| 24. | Min: | Minimum |
| 25. | Max: | Maximum |
| 26. | 1st Qu: | $1^{st}$ Quantile |
| 27. | 3rd Qu: | $3^{rd}$ Quantile |
| 28. | NSE: | Nigeria Stock Exchange |
| 29. | Rt: | Returns |
| 30. | Yt: | Present Value of Y at time t |
| 31. | yt-i: | Values of Y at lagged values at time t |
| 32. | F(y): | Cummulative Function |
| 33 | f(y): | Density Function |
| 35. | v: | Degree of freedom |
| 36. | $\gamma$: | Gamma |
| 37. | β: | Beta |
| 38. | $\alpha$: | Alpha |

39. $\xi$: Shape parameters

40. $\delta$: Delta

41. $\sum$: *Summation*

42. log: Logarithms

45. $l_t$: Log Likelihood function at time t

46. zt: Sequences

47. $\sigma$: sigma

48 . $\varepsilon$ : *Epsilon*

49. $\pi$: Pie

50. ht: Variance at time t

## 1.2 Aim and Objectives of the Study

The purpose of this study is to determine the best between GARCH (p, q) model and APARCH (p, q) model in the presence of asymmetric error innovations.

The specific objectives are to:

1. review GARCH models analytically
2. obtained Generalised Length Biased Scaled - t and Generalised Beta skew – t by remodifying Fisher concept of weighted distribution and McDonald Generalised Beta functions, respectively.

3. determine the framework of GARCH and APARCH models in the presence of newly proposed error innovations.

4. compare the forecast performance of the normal and non-normal error innovations.

5. recommend a model for an optimal forecast of the Nigeria Stock index using different model selection criteria.

## 1.3 Statement of the Problem

The disadvantage of the normal GARCH (p, q) model is that leverage outcome, volatility clustered and leptokurtosis cannot be captured adequately because both unconditional and conditional skewness are zero and conditional excess kurtosis is zero.This project attempts to redefine error innovations of GARCH (p, q) models based on violation of normality assumption for a non- normal distribution.

## 1.4 Motivation of the Study

Hitherto, most saturated probability models are based on assumptions of normal (symmetric) error innovation which have frequently produced low forecasting performance when one or more of these assumptions are violated. Thus, there is need to develop a more robust model capable of handling a non-normal (Asymmetric) error innovations as well as addressing issues of Leptokurtic and heavy tails.

## 1.5 Justification of the Study

Studies have shown that normality innovations in volatility models have lead to low forecast performance, clustered volatility and rejection of normality led us to the use of some heavy tailed distributions in modelling financial series. Researchers have attempted to fit the best GARCH models for share index of stock market assuming its error innovations to symmetrically distributed.

## 1.6 Significance of the Study

The work is of high significance, as it attempts to provide alternative means of estimation and forecast of asymmetric models of stock returns under violations of normal assumptions. A lot of works have been done on volatility models and its error terms in developed countries but such is scarce in Africa (Nigeria) markets.

## 1.7 Scope of Coverage

The research focused on development of newly proposed asymmetric error innovations for GARCH (p, q) and APARCH (p, q) models.

## 1.8 Numerical Illustration of the Proposed Models

The numerical illustration of these models was carried out on three different datasets: Nigeria Stock Exchange (NSE) monthly returns data between January 2000 to December 2015, Central Bank of Nigeria monthly shortfall excess credit data between October 2005 to December 2014, and simulated data of sizes 50, 100, 500 and 1000.

# Chapter Two

# LITERATURE REVIEW

## 2.1 Preamble

There have been  some empirical research chronicling the features of stock returns volatility in both emerging and developed stock market but such researches are not many for the Nigeria Stock Exchange (NSE). Symmetric GARCH models, Asymmetric GARCH models with their different error innovations were reviewed in this work. Conventional GARCH models assumed that the error innovations are normally distributed but recent studies shows that non-normal innovations are better to normal innovations. Stocks are generally non-normally distributed but skewed and also leptokurtic. The escalation of ARCH models is as a result of serial correlation in stock returns.

## 2.2 GARCH Models And It Extensions

The idea of ARCH has been around for some decades but it has its source in the olden days. The first to conduct a several study on characteristic of tentative prices is a French mathematician Bachelier (1900) who in that time applied Stochastic process (a Brownian's motion into mathematical finance) into theory of price speculation and then, there a long silence of period.

First to discover volatility clustering and thick tails in financial series is Mandelbrot (1963), he bring around the concern in the time series attributes of asset prices with his theory that random variables with an infinite population variance are absolutely indispensable for a workable analysis of changes in price. His observations such as small (greater) alterations tend to accompanied by small (greater) changes, unconditional distributions have thick tails and variances change over time are empirical facts for many economic and financial variable.

Aforementioned to the prelude of ARCH, researchers were very much cognisant of vicissitude in variance, just that informal processes were used to take record of this. For example, Mandelbrot (1963) used recursive estimates of the variance over time. ARCH model by Engle (1982) was the first formal model which seemed to depict the practical facts like conditional variances, Leptokurtosis etc.

The literature in ARCH is very wide, it is virtually impracticable to supply an extensive evaluation. Bollerslev (1986) introduce a Generalised ARCH (GARCH) model that supplies a frugal framework for the conditional variance which consummate this indispensability as it predicated on illimitable ARCH (∞) which diminishes the number of estimated parameters from illimitability to parsimonious two parameters. ARCH (p) and GARCH (p, q) models captured the conditional variance and leptokurtosis but their innovations is symmetric, ergo, they fail recedes in considering the leverage impact.

Other extension of non-linear GARCH models were proposed to address the quandary of leverage effect such as Asymmetric Power ARCH (APARCH) model by Glosten *et al*; (1993). Exponential GARCH (EGARCH) model by Nelson (1991), Ding e*t al;* (1993) introduce GJR – GARCH Model etc. Another dilemma encounter while utilising

GARCH models does not plenary embrace the fat tails property of high frequency financial time series. To surmount this predicament, Bollerslev (1989) and Beine et al; (2002) utilised the student's t- distribution.

In the same way, to capture skewness, Liu and Brosen (1995) utilized an asymmetric stable density. Fernandez and Steel (1998) provided a family of skewed distributions that capture skewness and kurtosis such as skewed t- distribution, Lambert and Laurent (2001) broaden these families of skewed distributions into GARCH models. Skewed generalised t- distribution was incorporated into GARCH and EGARCH models by Harris et al., (2004) to identify skewness and leverage effects of every day turnover and fit into international equity markets.

Loudon(2000), Pagan and Schwert (1990) independently carried out an extensive investigation on forecast conditional variance with asymmetric GARCH models which leads to better forecast performance of Asymmetric GARCH model to the symmetric types.

Dima *et al.*, (2008) explore the forecasting performance of volatility models such as EGARCH, GARCH, APARCH and GJR models together with their different error innovations such as normal, Student t and asymmetric student's t- distributions.

Yaya (2013) fitted volatility models for Nigeria Share Index using symmetric functional distributions.

Shittu et. al (2014) carried out a comparative study of empirical distributions for stock returns using Skewed–t, Beta skewed-t and Normal distributions.

## 2.3 Some Existing Empirical Distributions For Stock Returns.

Some of the Kurtic distributions commonly used in the statistics literature are reviewed in the subsections below.

### 2.3.1 Normal Distribution

Normal distribution is a continuous distribution which is derived as the limiting form of the binomial distribution for big value of n and p and q are not too small, given by below equation:

When n $\rightarrow\infty$, p $\neq$0 or 1

The pdf of normal distribution is

$$f(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(y-\mu)^2}{2\sigma^2}} \qquad 2.1$$

$-\infty < y < \infty$ , $-\infty < \mu < \infty$ , $\sigma > 0$

Where $\mu$ is the mean, $\sigma$ is the standard deviation $\pi \approx 3.14159$ , $e \approx 2.7182$

To compute probability of normal distribution we use

$$P(y_1 < y < y_2) = \int_{y_1}^{y_2} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(y-\mu)^2}{2\sigma^2}} dy \qquad 2.2$$

On substituting $z = \frac{y-\mu}{\sigma}$ in (1) we get

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} \qquad 2.3$$

Here mean = 0, standard deviation = 1. Equation 2.3 is known as standard form of normal distribution.

### 2.3.1.1 Moments Estimation of Normal Distribution

**Mean for Normal distribution**

$\mu = \int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{y^2}{2\sigma^2}}.y\,dy$ putting $\frac{y}{\sigma} = t$

$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-\frac{t^2}{2}} (t\sigma)(\sigma dt)$ 2.4

$$= \frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} t\, e^{-\frac{t^2}{2}} dt = \frac{\sigma}{\sqrt{2\pi}} \left[ e^{-\frac{t^2}{2}} \right]_{-\infty}^{+\infty}$$

$= \frac{\sigma}{\sqrt{2\pi}}[0] = 0$ 2.5

**Variance for Normal Distribution**

$$\mu_2^1 = \int y^2 . f(y)\sigma dy \qquad or\ \mu_2^1 = \int_{-\infty}^{+\infty} y^2 . \frac{1}{\sigma\sqrt{\pi}} e^{-} \frac{y^2}{2\sigma^2}\, dy$$

Putting $\frac{y^2}{2\sigma^2} = t$ or $y = \sqrt{2\sigma} t^{\frac{1}{2}}\, dy = \frac{\sqrt{2}\,\sigma\, dt}{2t^{\frac{1}{2}}}$

$\mu_2^1 = \int_{-\infty}^{+\infty} (2\sigma^2 t) \frac{1}{\sigma\sqrt{2\pi}} e^{-t} \left( \frac{\sqrt{2}\,\sigma\, dt}{2t^{\frac{1}{2}}} \right)$ 2.6

$$= \frac{2\sigma^2}{\sigma\sqrt{2\pi}} \frac{\sqrt{2}\sigma}{2} \int_{-\infty}^{+\infty} t^{\frac{3}{2}-1} e^{-t}\, dt$$

$$\left[\int_0^{\infty} y^{n-1}e^{-x}dy = \gamma(n)\,.\right]$$

$$= \frac{\sigma^2}{\sqrt{\pi}}\,.2\,\gamma\left(\frac{3}{2}\right) = 2\,\frac{\sigma^2}{\pi}\,.\frac{1}{2}\,\gamma\left(\frac{1}{2}\right) = \frac{\sigma^2}{\pi}\sqrt{\pi} = \sigma^2$$

$$\mu_2 = \mu_2^1 - (\mu_1)^2 = \sigma^2 - 0 = \sigma^2 \qquad 2.7$$

Variance = $\sigma^2$

## 2.3.1.2 Factorial Moment of Normal Distribution

The first factorial moment measure of a point process coincides with its first moment measure or intensity measure, which gives the expected or average number of points of the point process located in some region of space. In general, if the number of points in some region is considered as a random variable, then the moment factorial measure of this region is the factorial moment of this random variable.

$$\mu_{2n+1} = \int_{-\infty}^{+\infty}(y - \mu)^{2n+1}\, f(y)dy \qquad 2.8$$

$$\mu_{2n+1} = \frac{1}{\sigma\sqrt{2\pi}} = \int_{-\infty}^{+\infty}(y - \mu)^{2n+1}\, e^{-\frac{z^2}{2\sigma^2}}\, dy$$

$$\mu_{2n+1} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}(\sigma z)^{2n+1}\, e^{-\frac{z^2}{2}}\, dz\left[z = \frac{y-\mu}{\sigma}\right]$$

$$\mu_{2n+1} = \frac{\sigma^{2n+1}}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} z^{2n+1}\, e^{-\frac{z^2}{2}}\, dz$$

2.9

$\mu_{2n+1}$= 0 (since $z^{2n+1}e^{-\frac{z^2}{2}}$ is an odd function)

$$\mu_{2n} = \int_{-\infty}^{+=} (y - \mu)^{2n} \, f(y) dy$$

$\mu_{2n} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} (\sigma^2 z)^{2n} \, e^{-\frac{z^2}{2}} \, dz \; = \; \frac{\sigma^{2n}}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} z^{2n} \, e^{-\frac{z^2}{2}} \, dz$ 2.10

($z^{2n} e^{-\frac{z^2}{2}}$ is an even function)

$\mu_{2n} = \frac{2^n \sigma^{2n}}{\sqrt{\pi}} \int_0^{+\infty} z^{2n+1} \, e^{-t} \quad t\left(n - \frac{1}{2}\right) \, dt$ 2.11

$\mu_{2n} = \frac{2^n \sigma^{2n}}{\sqrt{\pi}} \gamma\left(\pi + \frac{1}{2}\right)$

Changing n to (n-1) we get

$\mu_{2n-2} = \frac{2^{n-1} \sigma^{2n-2}}{\gamma(\pi)} \, \gamma\left(n - \frac{1}{2}\right)$

On dividing, we get

$\frac{\mu_{2n}}{\mu_{2n-2}} = 2 \, \sigma^2 \, \frac{\gamma\left(\pi + \frac{1}{2}\right)}{\gamma\left(\pi - \frac{1}{2}\right)} = \frac{2 \, \sigma^2 \left(n - \frac{1}{2}\right) \gamma\left(\pi - \frac{1}{2}\right)}{\gamma\left(\pi - \frac{1}{2}\right)}$

$= 2 \, \sigma^2 \left(n - \frac{1}{2}\right)$

$$\mu_{2n} = \sigma^2 (2n - 1) \mu_{2n-2}$$

Which gives the recurrence relation for the moments of normal distribution

$$\mu_{2n} = [(2n - 1)\sigma^2][(2n - 3)\sigma^2]\mu_{2n-4}$$

$$\mu_{2n} = [(2n - 1)\sigma^2][(2n - 3)\sigma^2][(2n - 5)\sigma^2] \, \mu_{2n-6}$$

$$\mu_{2n} = [(2n - 1)\sigma^2][(2n - 3)\sigma^2][(2n - 5)\sigma^2] - 3 \, (3\sigma^2)(1 - \sigma^2) \, \mu_0$$

$$\mu_{2n} = (2n-1)(2n-3)(2n-5) - 1.\sigma^{2n}$$

=1.3.5.7… (2n -5) (2n-3) (2n -1) $\sigma^{2n}$ 2.12

## 2.3.2 The Logistic Distribution

Feller (1971) describe the Logistics distribution of a random variables as a continuous probability distribution given as :

$$f(y) = \frac{\exp\left(\frac{y-\mu}{\alpha}\right)}{\alpha\left[1+\exp\left(\frac{y-\mu}{\alpha}\right)\right]^2} \qquad 2.13$$

Where y is the random variable $(0 < y < \infty)$ With two parameter shapes, where the location and scale parameters are $\mu$ $(-\infty < \mu < \infty)$and $\alpha$ $(\alpha > 0)$, respectively. This distribution is more similar to normal distribution but it is characterized by thick tails and more suitable to model stock returns.

If $R_t$ follows a logistics distribution then

$$E(R_t) = \mu \ and \ Var(R_t) = \sigma^2 = \left(\pi^2/_3\right)\alpha^2 \qquad 2.14$$

#### 2.3.2.1 Moments of the Logistic Distribution

The method of moment will be used in obtaining the first moment about the origin and second moment about the mean of the distribution

$$E(y) = \int_0^{\infty} y f(y) \partial y$$

$$E(y) = \int_0^{\infty} y \frac{\exp\left(\frac{y-\mu}{\alpha}\right)\partial y}{\alpha\left[1+\exp\left(\frac{y-\mu}{\alpha}\right)\right]^2} \qquad 2.15$$

$$= \frac{1}{\alpha}\int_0^{\infty} y \frac{\exp\left(\frac{y-\mu}{\alpha}\right)}{\left[1+\exp\left(\frac{y-\mu}{\alpha}\right)\right]^2}\partial y$$

$$Let\ z = \exp\left(\frac{y-\mu}{\alpha}\right) \quad , \qquad \text{Inz} = \left(\frac{y-\mu}{\alpha}\right)$$

$$\frac{\partial z}{\partial y} = \frac{1}{\alpha}\exp\left(\frac{y-\mu}{\alpha}\right), \qquad \alpha \partial z = \exp\left(\frac{y-\mu}{\alpha}\right)\partial y$$

$$y - \mu = \alpha lnz \qquad \partial y = \frac{\alpha \partial z}{\exp\left(\frac{y-\mu}{\alpha}\right)}$$

$$y = \alpha lnz + \mu$$

$$E(y) = \frac{1}{\alpha}\int_0^{\infty} \frac{(\alpha lnz + \mu)z}{(1+z)^2}\alpha\frac{\alpha}{z}\partial z = \frac{1}{\alpha}\int_0^{\infty} x\frac{(\alpha lnz + \mu)z}{(1+z)^2}\alpha z\frac{\partial z}{z}$$

$$= \int_{-\infty}^{\infty} (\alpha lnz + \mu)(1+z)^{-2}\partial z$$

$$E(y) = \left[\int_0^{\infty} \alpha lnz(1+z)^{-2} + \mu\int_0^{\infty}(1+z)^{-2}\partial z\right] \qquad 2.16$$

$$= \frac{\mu(1+z)^{-2+1}}{-2+1}\bigg|_0^{\infty}$$

$$= \frac{\mu(1+z)^1}{-1}\Bigg|_0^\infty$$

$$= -\mu \frac{1}{1+z}\Bigg|_0^\infty$$

$$= -\mu\left(\frac{1}{1+\infty} - \frac{1}{1+0}\right) = -\mu(0-1)$$

$E(y) = \mu$ 2.17

As for the variance, we have

$$V(y) = E(y-\mu)^2 = \int_0^\infty (y-\mu)^2 f(y)\partial y$$

$= \int_0^\infty \frac{(y-\mu)^2 \exp\left(\frac{y-\mu}{\alpha}\right)\partial y}{\alpha\left[1+\exp\left(\frac{y-\mu}{\alpha}\right)\right]^2}$ 2.18

$$Let\ z = \frac{y-\mu}{\alpha} \quad , \qquad \frac{\partial z}{\partial y} = \frac{1}{\alpha}$$

$$y - \mu = z\alpha$$

$$(y-\mu)^2 = z^2\alpha^2$$

$$\partial y = \alpha\partial z$$

$$Var(y) = \int_0^\infty \frac{z^2\alpha^2\alpha}{\alpha(1+e^z)^2}\partial z$$

$$= \alpha^2 \int_0^\infty \frac{z^2 e^z}{\alpha(1+e^z)^2}\partial z$$

$$since \quad \int_0^\infty \frac{z^2 e^z}{\alpha(1+e^z)^2}\partial z = \frac{\pi^2}{3}$$

$$Var(y) = \alpha^2 \frac{\pi^2}{3} \qquad 2.19$$

### 2.3.2.2 Maximum Likelihood Estimate of Parameters

The parameters of the logistic distribution such as; $\alpha$ $and$ $\mu$ can be obtained using maximum likelihood estimate as follows:

$$L(y, \alpha) = \prod_{i=1}^{n} \frac{\exp\left(\frac{y-\mu}{\alpha}\right)}{\alpha\left[1+\exp\left(\frac{y-\mu}{\alpha}\right)\right]^2} \qquad 2.20$$

$$L(y, \alpha) = \frac{e^{\Sigma\left(\frac{y-\mu}{\alpha}\right)}}{y^n \prod_{i=1}^{n}\left[1+\mathrm{e}\left(\frac{y-\mu}{\alpha}\right)\right]^2} \qquad 2.21$$

The natural logarithm of equation 2.21 yields

$$\ln(y^i : \alpha) = lne^{\sum_{i=1}^{n}\left(\frac{y-\mu}{\alpha}\right)} - ln\alpha^n - ln\sum_{i=1}^{n}\left(1+\mathrm{e}\frac{y-\mu}{\alpha}\right)^2$$

$$= \sum_{i=1}^{n}\left(\frac{y-\mu}{\alpha}\right) - nln\alpha - \sum_{i=1}^{n} ln\left(1+\mathrm{e}\frac{y-\mu}{\alpha}\right)^2$$

$$= \sum_{i=1}^{n}\left(\frac{y-\mu}{\alpha}\right) - nln\alpha - -2\sum_{i=1}^{n} ln\left(1+\mathrm{e}\frac{y-\mu}{\alpha}\right)$$

$$ln\mathrm{L} = \alpha^{-1}\sum_{i=1}^{n}(y-\mu) - nln\alpha - -2\sum_{i=1}^{n} ln\left(1+\mathrm{e}\frac{y-\mu}{\alpha}\right) \qquad 2.22$$

Differentiating equation 2.22 and equate it derivative to zero yields

$$\frac{\partial lnL(y^i,\alpha,\mu)}{\alpha} = \alpha^{-2}\sum_{i=1}^{n}(y-\mu) - \frac{n}{\alpha} - \sum_{i=1}^{n}\frac{1}{\left(1+\mathrm{e}\frac{y-\mu}{\alpha}\right)}\frac{2}{\alpha}\,\mathrm{e}\frac{y-\mu}{\alpha}\left(1+\mathrm{e}\frac{y-\mu}{\alpha}\right) = 0$$

$$- \frac{\sum_{i=1}^{n}(y-\mu)}{\alpha^2} - \frac{n}{\alpha} - \frac{2}{\alpha}\sum_{i=1}^{n}\frac{e^{\frac{y-\mu}{\alpha}} - y}{\left(1+e^{\frac{y-\mu}{\alpha}}\right)^2} = 0$$

$$- \sum_{i=1}^{n}(y-\mu) - \frac{n}{\alpha} - \frac{2}{\alpha}\sum_{i=1}^{n}\frac{e^{\frac{y-\mu}{\alpha}}}{\left(1+e^{\frac{y-\mu}{\alpha}}\right)} = 0$$

$$- \sum_{i=1}^{n}(y-\mu) - \frac{n}{\alpha} - \frac{2}{\alpha}\sum_{i=1}^{n}\frac{e^{\frac{y-\mu}{\alpha}}}{\left(1+e^{\frac{y-\mu}{\alpha}}\right)} = 0$$

$$- \frac{\sum_{i=1}^{n}(y-\mu)}{} - n - 2\sum_{i=1}^{n}\frac{e^{\frac{y-\mu}{\alpha}} -}{\left(1+e^{\frac{y-\mu}{\alpha}}\right)} = 0$$

Differentiating equation (2) with respect to $\mu \; and$ set this to zero, we have;

$$\frac{\partial lnL(y^i,\alpha,\mu)}{\partial\mu} = -\alpha^{-1}\sum_{i=1}^{n}(y-\mu) - \frac{2}{\left(1+e^{\frac{y-\mu}{\alpha}}\right)}\sum_{i=1}^{n}\left(\frac{-1}{\alpha}\right)e^{\frac{y-\mu}{\alpha}} \quad 2.23$$

$$\frac{\partial lnL(y^i,\alpha,\mu)}{\partial\mu} = -\sum_{i=1}^{n}\frac{(y-\mu)}{\alpha} + \frac{2}{\alpha}\sum_{i=1}^{n}\left(\frac{e^{\frac{y-\mu}{\alpha}}}{1+e^{\frac{y-\mu}{\alpha}}}\right) = 0$$

$$\frac{2}{\alpha}\sum_{i=1}^{n}\left(\frac{e^{\frac{y-\mu}{\alpha}}}{1+e^{\frac{y-\mu}{\alpha}}}\right) = \sum_{i=1}^{n}\frac{(y-\mu)}{\alpha}$$

$$2\sum_{i=1}^{n}\left(\frac{e^{\frac{y-\mu}{\alpha}}}{1+e^{\frac{y-\mu}{\alpha}}}\right) = \sum_{i=1}^{n}(y-\mu) \quad 2.24$$

Equations (2) and (3) are called normal equations. There is no closed form solutions for theses equations hence they can be solve iteratively using numerical approach to obtain the MLE of $\alpha \; and \; \mu$.

## 2.3.3 The Student-t Distribution

A random variable y is said to be Student-t distribution if the probability density function is given as

$$f(y) = \frac{\gamma(\frac{v+1}{2})}{\gamma\left(\frac{v}{2}\right)\sqrt{\pi(v-2)\sigma^2}}\left[1 + \frac{(y-\mu)^2}{(v-2)\sigma^2}\right]^{-\frac{(v+1)}{2}} \quad 2.25$$

$\gamma(.)$ Is the gamma function, $\mu(-\infty < \mu < \infty)$ is the location parameter $, \sigma^2$ $(\sigma^2 > 0)$is the dispersion parameter and v (v>0) is a degree of freedom parameter and y $(0 < y < \infty)$ is the random variable . Peiro (1994) has accounted that student-t specification fit stock returns more better than many other competing ones.

### 2.3.3.1 Moment Of The Student-t Distribution

The method of moment will be used to obtain both the mean and variance of the random variable y with the distribution given above.

$$E(y) = \int_{-\infty}^{\infty} y\, f(y)\partial y$$

$$= \int_{-\infty}^{\infty} y \frac{\gamma(\frac{v+1}{2})}{\gamma\left(\frac{v}{2}\right)\sqrt{\pi(v-2)\sigma^2}}\left[1 + \frac{(y-\mu)^2}{(v-2)\sigma^2}\right]^{-\frac{(v+1)}{2}} \partial y \quad 2.26$$

$$= \beta \frac{1}{\left(\frac{1}{2}, \frac{v}{2}\right)} \int_0^{\infty} y(1+y)^{-\left(\frac{v+1}{2}\right)} + y^{-\frac{1}{2}}\partial y$$

$$y - \mu = y^{-\frac{1}{2}}(v-2)^{\frac{1}{2}}\sigma$$

$$y = y^{-\frac{1}{2}}(v-2)^{\frac{1}{2}}\sigma + \mu$$

$$E(y) = \beta \frac{1}{\left(\frac{1}{2}, \frac{v}{2}\right)} \int_0^{\infty} (y^{-\frac{1}{2}}(v-2)^{\frac{1}{2}}\sigma + \mu) y^{-\frac{1}{2}}(1+y)^{-\left(\frac{v+1}{2}\right)}\partial y \quad 2.27$$

$$= \beta \frac{1}{\left(\frac{1}{2}, \frac{v}{2}\right)} \int_0^\infty (v-2)^{\frac{1}{2}}(1+y)^{-\left(\frac{v+1}{2}\right)} + \mu \int y^{-\frac{1}{2}}(1+y)^{-\left(\frac{v+1}{2}\right)} \partial y$$

$$= \beta \frac{1}{\left(\frac{1}{2}, \frac{v}{2}\right)} \left(0 + \mu\beta\left(\frac{1}{2}, \frac{v}{2}\right)\right)$$

$=\frac{\mu\beta\left(\frac{1}{2},\frac{v}{2}\right)}{\beta\left(\frac{1}{2},\frac{v}{2}\right)}$

$E(y) = \mu$ 2.28

$E(y^2)= \beta \frac{1}{\left(\frac{1}{2},\frac{v}{2}\right)} \int_0^\infty (y^{\frac{1}{2}}(v-2)^{\frac{1}{2}}\sigma + \mu)^2 y^{-\frac{1}{2}}(1+y)^{-\left(\frac{v+1}{2}\right)} \partial y$ 2.29

$=\frac{\int_0^\infty (y^{\frac{1}{2}}(v-2)^{\frac{1}{2}}\sigma+\mu)^2 y^{-\frac{1}{2}}(1+y)^{-\left(\frac{v+1}{2}\right)} \partial y}{\beta\left(\frac{1}{2},\frac{v}{2}\right)}$

$= \frac{\int_0^\infty y(v-2)\sigma^2+2\sigma\mu y^{\frac{1}{2}}(v-2)^{\frac{i}{2}} y^{\frac{1}{2}}(1+y)^{-\left(\frac{v+1}{2}\right)} \partial y}{\beta\left(\frac{1}{2},\frac{v}{2}\right)}$

$=\frac{1}{\beta\left(\frac{1}{2},\frac{v}{2}\right)} \int_0^\infty y^{-\frac{1}{2}}(1+y)^{-\left(\frac{v+1}{2}\right)} \partial y + \mu^2 \int_0^\infty y^{\frac{1}{2}}(1+y)^{-\left(\frac{v+1}{2}\right)} \partial y + 2\mu\sigma(v-2)^{\frac{1}{2}} \int_0^\infty y^{-\frac{1}{2}}(1+$

$y)^{-\left(\frac{v+1}{2}\right)} \partial y$

$E(y^2)=\frac{(v-2)\sigma^2 \int_0^\infty y^{\frac{1}{2}}(1+y)^{-\left(\frac{v+1}{2}\right)} \partial y + \mu^2 \int_0^\infty y^{-\frac{1}{2}}(1+y)^{-\left(\frac{v+1}{2}\right)} \partial y}{\frac{1}{\beta\left(\frac{1}{2},\frac{v}{2}\right)}}$

Considering $\int_0^\infty y^{\frac{1}{2}}(1+y)^{-\left(\frac{v+1}{2}\right)} \partial y$

Recall that $\int_0^\infty y^{\frac{1}{2}}(1+y)^{-\left(\frac{v+1}{2}\right)} \partial y = \beta(\alpha, \beta)$

$$\alpha - 1 = \frac{1}{2}$$

$$\alpha = \frac{3}{2}$$

$$\alpha - \beta = \frac{v+1}{2}$$

$$\beta = \frac{v}{2} + \frac{1}{2} - \frac{3}{2} = \frac{v}{2} - \frac{2}{2}$$

$$\beta = \frac{v}{2} - 1$$

$$\int_0^\infty y^{\frac{1}{2}}(1+y)^{-\left(\frac{v+1}{2}\right)}\partial y = \beta\left(\frac{3}{2}, \frac{v}{2} - 1\right)$$

$$\beta\left(\frac{3}{2}, \frac{v}{2} - 1\right) = \frac{\gamma\left(\frac{3}{2}\right)\gamma\left(\frac{v}{2} - 1\right)}{\gamma\left(\frac{v}{2} + \frac{1}{2}\right)} = \frac{\gamma\left(\frac{3}{2}\right)\gamma\left(\frac{v}{2} - 1\right)}{\gamma\left(\frac{v+1}{2}\right)}$$

$$E(y^2) = \frac{(v-2)\sigma^2\beta\left(\frac{3}{2}, \frac{v}{2} - 1\right)}{\beta\left(\frac{v}{2} + \frac{1}{2}\right)} + \mu^2\frac{\beta\left(\frac{v}{2} + \frac{1}{2}\right)}{\beta\left(\frac{v}{2} + \frac{1}{2}\right)}$$

$$E(y^2) = \frac{(v-2)\sigma^2\gamma\left(\frac{3}{2}\right)\gamma\left(, \frac{v}{2} - 1\right)}{\left(\frac{v}{2} - \frac{1}{2}\right)\gamma\left(\frac{v}{2} - \frac{1}{2}\right)} + \mu^2$$

$$E(y^2) = \sigma^2 + \mu^2 \qquad 2.30$$

The Variance can be obtained using:

$$\sigma^2(y-\mu)^2 = \int_0^\infty (y-\mu)^2 f(y)\partial y$$

$$= \int_0^\infty (y-\mu)^2 \frac{\gamma(\frac{v+1}{2})}{\gamma\left(\frac{v}{2}\right)\gamma\left(\frac{1}{2}\right)(v-2)^{-\frac{1}{2}}\sigma}\left[1 + \frac{(y-\mu)^2}{(v-2)\sigma^2}\right]^{-\frac{(v+1)}{2}}\partial y \qquad 2.31$$

$$V(y) = \frac{1}{\beta\left(\frac{1}{2}, \frac{v}{2}\right)}\int_0^\infty (y-\mu)^2\, y^{-\frac{1}{2}}(1+y)^{-\left(\frac{v+1}{2}\right)}\partial y$$

Recall;$(y-\mu)^2 = y(v-2)\sigma^2$

$= \frac{1}{\beta\left(\frac{1}{2},\frac{v}{2}\right)}\int_0^\infty y(v-2)\sigma^2\, y^{-\frac{1}{2}}(1+y)^{-\left(\frac{v+1}{2}\right)}\partial y$

$$V(y) = \frac{(v-2)\sigma^2}{\beta\left(\frac{1}{2},\frac{v}{2}\right)}\int_0^\infty y^{\frac{1}{2}}(1+y)^{-\left(\frac{v+1}{2}\right)}\partial y$$

Recall: $\beta(\alpha,\beta) = \int_0^\infty y^{\alpha-1}(1+y)^{-\left(\frac{v+1}{2}\right)}\partial y$

$$\alpha - 1 = \frac{1}{2}$$

$$\alpha = \frac{3}{2}$$

$$\alpha - \beta = \frac{v+1}{2}$$

$$\beta = \frac{v}{2} + \frac{1}{2} - \frac{3}{2} = \frac{v}{2} - \frac{2}{2}$$

$$\beta = \frac{v}{2} - 1$$

$V(y) = \frac{\sigma^2(v-2)}{\beta\left(\frac{1}{2},\frac{v}{2}\right)}\beta\left(\frac{3}{2},\frac{v}{2}-1\right)$ 2.32

$$V(y) = \sigma^2(v-2)\frac{\gamma\left(\frac{3}{2}\right)\gamma\left(\frac{v}{2}-1\right)}{\gamma\left(\frac{v}{2}+\frac{1}{2}\right)}\frac{\gamma\left(\frac{v}{2}+\frac{1}{2}\right)}{\gamma\left(\frac{v}{2}\right)\gamma\left(\frac{1}{2}\right)}$$

$$= \frac{\sigma^2(v-2)\frac{1}{2}\gamma\left(\frac{1}{2}\right)\gamma\left(\frac{v}{2}-1\right)}{\gamma\left(\frac{1}{2}\right)\left(\frac{v}{2}-1\right)\gamma\left(\frac{v}{2}-1\right)}$$

$$= \frac{\sigma^2(v-2)}{2\left(\frac{v}{2}-1\right)}$$

$$= \frac{\sigma^2(v-2)}{(v-2)}$$

$$V(y) = \sigma^2 \qquad 2.33$$

#### 2.3.3.2 Verification Of True Pdf Of Student-t Distribution:

$$f\left(y\right)=\frac{\Gamma\left(\frac{v+1}{2}\right)}{\sigma\Gamma\left(\frac{v}{2}\right)\sqrt{\pi(v-2)}}\int_{-\infty}^{\infty}\left[1+\left(\frac{y-\mu}{\sigma}\right)^{2}\frac{1}{v-2}\right]^{-\frac{v+1}{2}}dy \qquad 2.34$$

Let $Z=\frac{y-\mu}{\sigma}$ , $\frac{dz}{dy}=\frac{1}{\sigma}\Rightarrow dy=\sigma dz$

$$\frac{\Gamma\left(\frac{v+1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)\sqrt{\pi(v-2)}}\int_{-\infty}^{\infty}\left[1+\frac{Z^{2}}{v-2}\right]^{-\frac{v+1}{2}}dz \qquad 2.35$$

Let m= $\frac{Z^{2}}{v-2}$ , $\frac{dm}{dz}=\frac{2Z}{v-2}\Rightarrow dz=\frac{v-2}{2Z}dm=\frac{v-2}{2m^{\frac{1}{2}}(v-2)^{-\frac{1}{2}}}dm$

$$\frac{\Gamma\left(\frac{v+1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)\Gamma\left(\frac{1}{2}\right)(v-2)^{\frac{1}{2}}}\int_{-0}^{\infty}[1+m]^{-\frac{v+1}{2}}\frac{v-2}{2m^{\frac{1}{2}}(v-2)^{-\frac{1}{2}}}dm$$

$$\frac{\Gamma\left(\frac{v+1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)\Gamma\left(\frac{1}{2}\right)}\int_{-0}^{\infty}[1+m]^{-\frac{v+1}{2}}m^{-\frac{1}{2}}dm \qquad 2.36$$

Recall from beta function of the second kind that

$$\beta(\alpha,\beta)=\int_{0}^{\infty}m^{\alpha-1}(1+m)^{-(\alpha+\beta)}dm \qquad 2.37$$

$$\alpha - 1 = -\frac{1}{2} \quad \alpha + \beta = \frac{v+1}{2}$$

$$\alpha = \frac{1}{2} \qquad \beta = \frac{v+1}{2} - \frac{1}{2} = \frac{v}{2}$$

$$\frac{\Gamma\left(\frac{v+1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)\Gamma\left(\frac{1}{2}\right)} \beta\left(\frac{1}{2}, \frac{v}{2}\right)$$

$$\frac{\Gamma\left(\frac{v+1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)\Gamma\left(\frac{1}{2}\right)} \frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{v}{2}\right)}{\Gamma\left(\frac{v+1}{2}\right)} = 1 \qquad 2.38$$

This verified that student t is a true pdf.

### 2.3.3.3 CUMULATIVE DENSITY OF STUDENT –T DISTRIBUTION

The cumulative density function of a random variable Y may be define as the probability that the random variable Y takes a value less than or equal to y. Mathematically

$$F(y) = P(Y \leq y) \qquad , \quad 0 \leq F(y) \leq 1$$

Hence the Cumulative density of Student t can be express as:

$$F(y) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)\Gamma\left(\frac{1}{2}\right)} \int_0^y \left[1+m\right]^{-\frac{v+1}{2}} m^{-\frac{1}{2}} dm \qquad 2.39$$

$$= \frac{\Gamma\left(\frac{v+1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)\Gamma\left(\frac{1}{2}\right)} \beta(m; \frac{1}{2}, \frac{v}{2}) \qquad 2.40$$

$$= \frac{\beta(m; \frac{1}{2}, \frac{v}{2})}{\beta(\frac{1}{2}, \frac{v}{2})} = I \qquad 2.41$$

Where $m = \frac{Z^2}{v-2} = \frac{\left(\frac{y-\mu}{\sigma}\right)^2}{v-2}$

$$F(y) = I = \frac{\beta\left[\left(\frac{y-\mu}{\sigma}\right)^2 \frac{1}{v-2}, \frac{1}{2}, \frac{v}{2}\right]}{\beta\left(\frac{1}{2}, \frac{v}{2}\right)} \qquad 2.42$$

Hence, F(y) is the cumulative distribution function.

# Chapter Three

# METHODOLOGY

## 3.1 Preamble

Autoregressive Conditional Heteroskedasticity (ARCH) models are financial time series instrument utilised to calculate and project stock returns volatility. ARCH (q) model is the first formal model captured and explained volatility of time series data. Generalised ARCH (GARCH) model addressed the problem of over parameterization and characterised the order of parameter from infinity to two. In this section different ARCH models will be review and its properties. Some of the ARCH models considered in this work are ARCH (1) model, GARCH (1,1) model and APARCH (1,1) model. The innovations that captures information in series are discussed, some of the trending conventional error innovations considered in this work are Normal, student-t and Generalised Error Distribution (GED). Newly error innovations were developed into volatility models such as Generalised Beta Skewed-t and Generalised Length Biased Scaled-t distribution, they were obtained by remodifying McDonald Link Function and Fishers Weighted Functions respectively. Unlike general method of estimating linear models, the log-likelihood captures the information and Quasi- Maximum Likelihood method was used to estimate the parameters.

## 3.2 ARCH (q) Model

ARCH (q) model is the first formal volatility model that captures unequal changing variance in time series regularities. Engle (1982) proposed an Autoregressive Conditional Heteroskedasticity (ARCH) processes using lagged error terms to model time varying conditional variance. Let $y_t$ be a series of returns or mean equation given as:

$$Y_t = E(y_t/I_{t-i}) + \boldsymbol{\epsilon_t} \qquad 3.1$$

From Equation 3.1, $I_{t-1}$ represent information sets available at time t-i and $\varepsilon_t$ are the random innovation (surprise) . The expected error term is

$$\mathbf{E(\boldsymbol{\epsilon_t})=0} \qquad 3.2$$

In solving conditional variance dynamics, simple quadratic function of the lagged values of the innovations was used to estimate the variance of returns as proposed by Engle:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \, \epsilon_{t-1}^2 \qquad 3.3$$

Where $\varepsilon_t = \sigma_t z_t$ and $z_t$ is identical independent random variable with mean zero and variance one.

## 3.3 ARCH Models Properties

In this section, evidence based properties of ARCH model would be discussed bearing in mind that ARCH process of deterministic and stochastic processes.

### 3.3.1 The Conditional Mean and Variance

Let $F_{t-1}$ represent information set at time t-1, which can be define as

$$F_{t-1} = \sigma\left\{\varepsilon_t, -\infty < i \le t-1\right\} \qquad 3.4$$

In any model in which $\sigma_t$ is determined in connection to $F_{t-1}$

$$\begin{aligned} & E\left(y_t\right) = E\left(\sigma_t \varepsilon_t\right) = E\left(E\left(\sigma_t \varepsilon_t \left| F_{t-1}\right.\right)\right) \\ & = E\left(\sigma_t E\left(\varepsilon_t \left| F_{t-1}\right.\right)\right) = E\left(\sigma_t E\left(\varepsilon_t\right)\right) \\ & = \left(\sigma_t .0\right) = 0 \end{aligned} \qquad 3.5$$

**The Conditional Variance of $\varepsilon_t$ is**

$$\sigma^2 = Var\left(\varepsilon_t / F_{t-1}\right) \qquad 3.6$$

from the concept of $\sigma^2 = E(X-\mu)^2 / F_{t-1}$ 3.7

$$= E\left(\varepsilon_t - E(\varepsilon_t) / F_{t-1}\right)^2 / F_{t-1}$$

$$= E\left(\varepsilon_t^2 - E(\varepsilon_t) / F_{t-1}\right) \text{ Since } E\left(\varepsilon_t / F_{t-1}\right) = 0$$

$$= E\left(\varepsilon_t \varepsilon_t^2 / F_{t-1}\right)$$

$$= E\left(\varepsilon_t^2\right)E(\alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\varepsilon_{t-2}^2 + ... + \alpha_m\varepsilon_{t-m}^2 / F_{t-1})$$

Since $Var(\varepsilon_t^2) = E\left(\varepsilon_t^2\right) = 1$

$$= \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\varepsilon_{t-2}^2 ... + \alpha_m\varepsilon_{t-m}^2$$

$$= \sigma_t^2 \qquad 3.8$$

In order to comprehend the ARCH model, it is good to painstakingly examined the ARCH (1) model so that generalisation can be made from it;

$$\varepsilon_t = \sigma_t z_t \qquad 3.9$$

$$\sigma_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 \qquad 3.10$$

where $\alpha_0 > 0$ and $\alpha_1 \geq 0$ are condition for non- negative conditional variance and stationarity.

## 3.3.2 Unconditional Mean And Variance

First, the unconditional mean of $\varepsilon_t$

$$E(\varepsilon_t) = E\left(\varepsilon_t / F_{t-1}\right) == E(\sigma_t)E(\varepsilon_t) = 0 \qquad 3.11$$

Variance of $\varepsilon_t$ $Var(\varepsilon_t) = E(\varepsilon_t^2) = E\left[\varepsilon_t^2 / F_{t-1}\right]$ 3.12

$$= E(\alpha_0 + \alpha_1 \varepsilon_{t-1}^2)$$

$$= \alpha_0 + \alpha_1 E(\varepsilon_{t-1}^2) \qquad 3.13$$

Because $\varepsilon_t$ is stationary process with $E(\varepsilon_t) = 0$

$$Var(\varepsilon_t) = Var(\varepsilon_{t-1}) = E(\varepsilon_{t-1}^2) \qquad 3.14$$

i.e $Var(\varepsilon_t) = \alpha_0 + \alpha_1 Var(\varepsilon_t)$

$$\Rightarrow Var(\varepsilon_t) - \alpha_1 Var(\varepsilon_t) = \alpha_0$$

$$\Rightarrow Var(\varepsilon_t)\left[1 - \alpha_1\right] = \alpha_0$$

$$\Rightarrow Var(\varepsilon_t) = \frac{\alpha_0}{1 - \alpha_1} \qquad 3.15$$

To attain positive variance of $\varepsilon_t$ we assume $0 \le \alpha_1 \le 1$. Therefore, unconditional variance is constant over time, In general

$$Var(\varepsilon_{t-1}) = Var(\varepsilon_{t-m}) = Var(\varepsilon_t) = \frac{\alpha_0}{1 - \alpha_1 - \alpha_2 - \ldots - \alpha_m} \qquad 3.16$$

It is require that $\alpha_0 > 0$ and $\alpha_1 + \alpha_2 + \ldots + \alpha_m < 1$ for ratio to be finite and positive.

Since $E(\varepsilon_t / F_{t-1}) = 0$, so $E(\varepsilon_t \varepsilon_{t-m}) = 0$, i=1, … m. This verify that $\varepsilon_t$ is nothing but the White noise process. All the roots of $1 - \alpha_1 z - \alpha_2 z^2 - ... - \alpha_m z^m = 0$ must lie outside the unit circle.

### 3.3.3 Fourth Moments Of ARCH(1) Model

Empirically speaking financial series tails is not normally distributed, hence there exist more than the first and second moment. In a lot of applications, higher moments of the $\varepsilon_t$ must exist, $\alpha_1$ must also satisfy some additional constrains in order to know whether $\varepsilon_t$ will always follow assumption of normality with or in the present of heavy tail behaviour. So the fourth moment of $\varepsilon_t$ must be <3 infinite if it going to absorb normality, otherwise, it the fourth moment of $\varepsilon_t$ is >3 or finite heavily tail behaviour affects.

So

$$E(\varepsilon_t^4) = 3E\left(\sigma_t^4\right) \quad 3.17$$

$$E(\varepsilon_t^4) = E(\varepsilon_t^4 / F_{t-1}) = 3E(\alpha_0 + \alpha_1 \varepsilon_{t-1}^2)^2$$

$$= 3E(\alpha_0^2 + 2\alpha_0 \alpha_1 \varepsilon_{t-1}^2 + \alpha_1^2 \varepsilon_{t-1}^4) \quad 3.18$$

.

If $\varepsilon_t^4$ is the fourth order stationary let $M_4 = E(\varepsilon_t^4)$ then we have

$$m_4 = 3(\alpha_0^2 + 2\alpha_0 \alpha_1 Var(\varepsilon_t) + \alpha_1^2 m_4) \quad 3.19$$

$$= 3\alpha_0^2\left(1 + 2\frac{\alpha_1}{1-\alpha_1}\right) + 3\alpha_1^2 m_4$$

$$= 3\alpha_0^2\left(1 + \frac{2\alpha_1}{1-\alpha_1}\right) + 3\alpha_1^2 m_4$$

$$= 3\alpha_0^2\left(\frac{1-\alpha_1+2\alpha_1}{1-\alpha_1}\right) + 3\alpha_1^2 m_4$$

$$m_4 = \frac{3\alpha_0^2(1+\alpha_1)}{1-\alpha_1} + 3\alpha_1^2 m_4$$

$$m_4 - 3\alpha_1^2 m_4 = \frac{3\alpha_0^2(1+\alpha_1)}{1-\alpha_1}$$

$$m_4(1-3\alpha_1^2) = \frac{3\alpha_0^2(1+\alpha_1)}{1-\alpha_1}$$

$$m_4 = E\left(\varepsilon_t^4\right) = \frac{3\alpha_0^2(1+\alpha_1)}{(1-\alpha_1)(1-3\alpha_1^2)} \qquad 3.20$$

Fourth moment of $\varepsilon_t$ must satisfy the condition $1-3\alpha_1^2 > 0$ ie $0 \le \alpha_1^2 \le \frac{1}{3}$

Finally, the Kurtosis of $\varepsilon_t$ is:

$$k(x) = \frac{E(X-\mu_x)^4}{\sigma_x^4} = \frac{\mu_4}{\sigma_x^4} = \frac{E(\varepsilon_t^4)}{\left[Var(\varepsilon_t)\right]^2} = \frac{\dfrac{3\alpha_0^2(1+\alpha_1)}{(1-\alpha_1)(1-3\alpha_1^2)}}{\dfrac{\alpha_0^2}{\left(1-\alpha_1\right)^2}} \qquad 3.21$$

$$= \frac{3\alpha_0^2(1+\alpha_1)}{(1-\alpha_1)(1-3\alpha_1^2)} x \frac{\left(1-\alpha_1\right)^2}{\alpha_0^2}$$

$$= \frac{3(1+\alpha_1)}{(1-3\alpha_1^2)}\left(1-\alpha_1\right)$$

$$= 3\frac{1+\alpha_1^2}{1-3\alpha_1^2} \qquad 3.22$$

From equation (3.20) there is excess kurtosis, leading to positive $\varepsilon_t$ with thick tail distribution. Recall that the excess kurtosis of normal distribution is zero. The excess kurtosis is $k(x)-3$ and normally distributed innovation is $k(x)=3$.

## 3.4 Limitation Of ARCH Model

1. High order of q is required to capture volatility process in ARCH models.
2. The error innovation is assumed to be symmetric, they failed to capture leverage effect

## 3.5 GARCH (p, q) Model

Bollerslev (1986) generalised Engle's model to make it more realistic; the generalisation was named "GARCH", the most common used financial time series model and has tons of more sophisticated models. The Generalized ARCH (GARCH) model is the specification of ARCH model which lessen the number of calculated framework from infinity to two. In other word, this model address issues of over parameterization thereby makes it more parsimony and flexible to use. The GARCH (p,q) model is defined by

$$Y_t = u_t + \varepsilon_t \qquad 3.23$$

$$\varepsilon_t = z_t \sqrt{\sigma_t^2} \qquad 3.24$$

Where $\varepsilon_t$ is white noise, $z_t$ is sequence of identical independent random variable and

$\sigma_t^2$ is

the conditional time variance.

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \qquad 3.25$$

Where $\alpha_0$ , $\alpha_i$ and $\beta_j$ are the parameters to be calculated for sufficiency, positivity in conditional variance $\alpha_0 > 0$ , $\alpha_i \geq 0$ (for i=1, ---, q) , $\beta_j \geq 0$ (for j=1, …, p) and the innovation sequence ($z_t$ ) is independent and identically distributed with mean =0 and unit variance 1.

The idea is that, the conditional sequence of $y_t^2$ given information available up to time t-1, has an autoregressive structure and is positively correlated to its own recent past and to recent values of the squared returns $y_t^2$. Then captures the idea of conditional variance, also being persistent: large or small values of $y_t^2$ are likely to be followed by large or small values.

When lag operator is L, the variance is :

$$\sigma_t^2 = \alpha_0 + \alpha(L)\epsilon_t^2 + \beta(L)\sigma_t^2 \qquad 3.26$$

Where $\alpha(L) = \sum_{i=1}^{q} \alpha_i L^i$ and $\beta(L) = \sum_{j=1}^{p} \beta_j L^j$

If all the roots of the polynomial $|1-\beta(L)| = 0$ lies outside the unit circle, we have

$$\sigma_t^2 = \alpha_0 (|1-\beta(L)|)^{-1} + \alpha_i(L)(|1-\beta(L)|)^{-1}\epsilon_t^2 \qquad 3.27$$

Equation 27 shows that the conditional variance relies on linearly on all previous squared residuals, therefore it is seen as ARCH(∞) process. The conditional variance of $y_t$ can become larger than the unconditional variance. if past realizations of $\epsilon_t^2$ are larger than $\sigma^2$ the unconditional variance is given by:

$$\sigma^2 = \frac{\alpha_0}{1-\sum_{i=1}^{q}\alpha_i-\sum_{j=1}^{p}\beta_j} \qquad 3.28$$

## 3.6 Properties Of GARCH Model

Since Generalised ARCH model of Bollerslev is volatility model that address the drawback of of ARCH process, some interesting properties of GARCH (p,q) models are given below:

## 3.6.1 Unconditional Mean And Variance

The unconditional mean is Zero

Define the information set :

$$
\begin{aligned}
E(y_t) &= E(\sigma_t \varepsilon_t) = E\left[E(\sigma_t \varepsilon_t | F_{t-1})\right] \\
&= E(\sigma_t E(\varepsilon_t | F_{t-1})) = E(\sigma_t .0) = 0
\end{aligned}
\qquad 3.29
$$

The unconditional variance in equation (3.28) can be derived as follows

First and foremost to compute $E(\varepsilon_t^2)$, it is helpful to consider an alternative representation of $\varepsilon_t^2$ by defining the sequence

$$Z_t = \varepsilon_t^2 - \sigma_t^2 \qquad 3.30$$

$Z_t$ can be treated as white noise, it has been proved that $Z_t$ is martingale difference and therefore has mean zero ie the lack of serial correlation property is more tricky as it would required $E(\varepsilon_t^4) = \infty$, which is not always true. We now proceed with alternative representative

$$\varepsilon_t^2 = \sigma_t^2 + Z_t$$

$$= \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_t^2 \sigma_{t-j}^2 + Z_t$$

$$= \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_t^2 \sigma_{t-j}^2 + Z_t$$

$$= \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_t^2 \sigma_{t-j}^2 - \sum_{j=1}^{q} \beta_t^2 Z_{t-j}^2 + Z_t$$

If we denote R=Max(p,q), $\alpha_i = 0$ for i> p and $\beta_i = 0$ for j > q, then the above can be written as

$$\varepsilon_t^2 = \alpha_0 + \sum_{i=1}^{R} (\alpha_i + \beta_i)\varepsilon_{t-1}^2 - \sum_{j=1}^{q} \beta_j z_{t-j} + z_t \qquad 3.31$$

Using Stationarity (which implies $E(\varepsilon_t^2) = E(\varepsilon_{t+h}^2)$ the unconditional variance is now easy to obtain:

$$E(\varepsilon_t^2) = \alpha_0 + \sum_{i=1}^{R} (\alpha_i + \beta_i)\varepsilon_{t-1}^2 - \sum_{j=1}^{q} \beta_j E(z_{t-j}) + E(z_t)$$
$$= \alpha_0 + E(\varepsilon_t^2) \sum_{i=1}^{R} (\alpha_i + \beta_i)\varepsilon_{t-1}^2$$

Which gives

$$E(\varepsilon_t^2) = \frac{\alpha_0}{1 - \sum_{i=1}^{R} (\alpha_i + \beta_i)} \qquad 3.32$$

Like ARCH, $\alpha_0 > 0$ ,$\alpha_i \geq 0$ (for i=1, ---, q) and $\beta_j \geq 0$ (for j=1, …, p) is sufficient for GARCH to be non-negative .

## 3.6.2 Conditional Mean and Variance

Let $F_{t-1}$ denote the information set available at time $t-1$. The conditional mean of $\varepsilon_t^2$ is

$$E(\varepsilon_t / F_{t-1}) = \sqrt{h_t}.E(\varepsilon_t / F_{t-1}) = 0 \qquad 3.33$$

From 25, it implies that the conditional variance of $\varepsilon_t$ is

$$\sigma_t^2 = Var\left(\varepsilon_t / F_{t-1}\right)$$
$$= E\left\{\left[\varepsilon_t - E\left(\varepsilon_t / F_{t-1}\right)\right]^2 / F_{t-1}\right\}$$
$$= E\left(\varepsilon_t^2 / F_{t-1}\right)$$

Sin *ce*

$$E\left(\varepsilon_t / F_{t-1}\right) = 0$$
$$= E\left(\sigma_t^2 \varepsilon_t^2 / F_{t-1}\right)$$
$$= E\left(\varepsilon_t^2\right) E(\alpha_0 + \sum_{i=i}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 / F_{t-1}) \quad = \sigma_t^2 \qquad 3.34$$
$$= \alpha_0 + \sum_{i=i}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$$

### 3.6.3 Coefficients of a Conditional Gaussian GARCH (1,1) Model

Focusing on the simplest GARCH (1,1) model, weakness and strength of GARCH model can be seen:

$$\varepsilon_t = \sqrt{\sigma_t^2} z_t$$
$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \qquad 3.35$$

With. $z_t \sim N(0,1)$, a large $\varepsilon_{t-1}^2$ leads to a large $\sigma_t^2$ which implies that a large $\varepsilon_{t-1}^2$ tends to be followed by another large $\varepsilon_t^2$, the well-known phenomenon of volatility clustering in financial time series.

It can be proven that if $1 - 2\alpha_1 - \left(\alpha_1 + \beta_1\right)^2 > 0$ then

$$\frac{E\left(\varepsilon_t^4\right)}{\left[E\left(\varepsilon_t^2\right)\right]^2} = \frac{3\left[1 - \left(\alpha_1 + \beta_1\right)^2\right]}{1 - 2\alpha_1 - \left(\alpha_1 + \beta_1\right)^2} > 3 \qquad 3.36$$

Relative to ARCH models, the tail distribution of a GARCH (1,1) process is fattier than that of a normal distribution. Also, the model provides a simple parametric function that can be used to explained the volatility evolution. Recall from equation (3.27) by definition

$$E\left(\varepsilon_t^2 / F_{t-1}\right) = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2$$

And therefore

$$E\left(\varepsilon_t^4 / F_{t-1}\right) = 3\left[E\left(\varepsilon_t^2 / F_{t-1}\right)^2\right] = 3\left(\alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2\right)^2 \qquad 3.37$$

Hence

$$E\left(\varepsilon_t^4\right) = 3E\left(\alpha_0^2 + \alpha_1^2\varepsilon_{t-1}^4 + \beta_1^2\sigma_{t-1}^2 + 2\left(\alpha_0\alpha_1\varepsilon_{t-1}^2\right)\right) + 2\left(\alpha_0\beta_1\sigma_{t-1}^2\right) + 2\left(\alpha_1\beta_1\varepsilon_{t-1}^2\sigma_{t-1}^2\right)$$

*Denote*

$$E\left(\varepsilon_t^4\right) = m_4$$

Note that

$$E\left(\varepsilon_t^4\right) = 3E\left(\alpha_0^2 + \alpha_1^2\varepsilon_{t-1}^4 + \beta_1^2\sigma_{t-1}^2 + 2\left(\alpha_0\alpha_1\varepsilon_{t-1}^2\right)\right) + 2\left(\alpha_0\beta_1\sigma_{t-1}^2\right) + 2\left(\alpha_1\beta_1\varepsilon_{t-1}^2\sigma_{t-1}^2\right)$$

*Denote*

$$E\left(\sigma_t^2\right) = E\left[E\left(\varepsilon_t^2 / F_{t-1}\right)\right] = E\left(\varepsilon_t^2\right) = \sigma^2$$

$$E\left[\left(\sigma_t^2\right)^2\right] = E\left[E\left(\varepsilon_t^2 / F_{t-1}\right)^2\right] = \sigma^4 = \frac{m_4}{3}$$

$$E\left[\left(\varepsilon_{t-1}^2\sigma_{t-1}^2\right)^2\right] = E\left[E\left(\varepsilon_{t-1}^2\sigma_{t-1}^2 / F_{t-1}\right)\right] = E\left[\varepsilon_{t-1}^2 E\left(\sigma_{t-1}^2 / F_{t-1}\right)\right]$$

$$= E\left(\varepsilon_{t-1}^2\left(E\left(\varepsilon_{t-1}^2 / F_{t-2}\right) / F_{t-1}\right)\right)$$

$$= E\left(\varepsilon_{t-1}^2(E\left(\varepsilon_{t-1}^2 / F_{t-2}\right)\right)$$

$$= \sigma^4 = \frac{m_4}{3}$$

By expansion

*Consequently*

$$\left(1-3\alpha_1^2-\beta_1^2-2\alpha_1\beta_1\right)m_4=3\alpha_0^2+6\alpha_0\alpha_1\sigma^2+6\alpha_0\beta_1\sigma$$

$$=3\alpha_0^2++\frac{6\alpha_0\alpha_1.\alpha_0}{\left(1-\alpha_1-\beta_1\right)}+\frac{6\alpha_0\beta_1.\alpha_0}{\left(1-\alpha_1-\beta_1\right)}$$

$$=\frac{3\alpha_0^2\left(1-\alpha_1-\beta_1\right)+6\alpha_0^2\alpha+6\alpha_0^2\beta_1}{\left(1-\alpha_1-\beta_1\right)}$$

$$=\frac{3\alpha_0^2\left(1+\alpha_1+\beta_1\right)}{\left(1-\alpha_1-\beta_1\right)}$$

That is,

$$m_4=\frac{3\alpha_0^2\left(1+\alpha_1+\beta_1\right)}{\left(1-\alpha_1-\beta_1\right)(1-3\alpha_1^2-\beta_1^2-2\alpha_1\beta_1)}$$

The unconditional Kurtosis of $\varepsilon_t$ is

$$\frac{m_4}{\left(\sigma^2\right)^2}=\frac{3\alpha_0^2\left(1+\alpha_1+\beta_1\right)}{\left(1-\alpha_1-\beta_1\right)(1-3\alpha_1^2-\beta_1^2-2\alpha_1\beta_1)}\,x\,\frac{\left(1-\alpha_1-\beta_1\right)^2}{\alpha_0^2}$$

$$\frac{m_4}{(\sigma^2)^2}=\frac{3\left(1-\left(\alpha_1+\beta_1\right)^2\right)}{1-\left(\alpha_1+\beta_1\right)^2-2\alpha_1^2}>3 \qquad 3.38$$

Which implies excess leptokurtosis.

### 3.6.4 Uniqueness and Stationarity of GARCH (p,q) Model

The Stationarity and uniqueness of GARCH (p,q) model can be attained if and only if

$$:\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1 \qquad 3.39$$

### 3.6.5 The Conditional Density of GARCH (p,q) Model

Assuming that $z_t$ is a Gaussian variate, the conditional density of $\varepsilon_t$ given all the information update to $t-1$ is

$$f(\varepsilon_t/F_{t-1}) = \sqrt{\sigma_t^2}\, F(\varepsilon_t/F_{t-1}) = \sqrt{\sigma_t^2}\ .\ \mathrm{N}(0, 1) \sim N(0\,, \sigma_t^2\,) \qquad 3.40$$

## 3.7 Estimation Of GARCH (p, q) Model With Normal Error Innovation

$$y_t = \alpha_0 + \sum \alpha_i y_{t-i} + \varepsilon_t \qquad 3.41$$

$$h_t = \sigma_t^2 = \sum_{t=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{t=i}^{q} \beta_i \sigma_{t-i}^2 \qquad 3.42$$

If $y_t \sim N(\mu\,, \sigma_t^2)$

$$f(y_t) = \frac{1}{\sqrt{2\pi\sigma_t^2}} e^{-\frac{1}{2}\left(\frac{y_t-\mu}{\sigma_t}\right)^2} \qquad 3.43$$

$$= \left(2\pi h_t\right)^{-\frac{n}{2}} e^{-\frac{1}{2}\sum_{t=1}^{n}\left(\frac{y_t-\mu}{h_t}\right)^2}$$

$$= \left(2\pi h_t\right)^{-\frac{n}{2}} e^{-\frac{1}{2}\sum_{t=1}^{n}\frac{\varepsilon_t^2}{h_t}} \qquad 3.44$$

Consider the pdf of Normal distribution of normal random variable is given as

$$f(R_t) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{R_t-\mu}{\sigma^2}\right)} \qquad 3.45$$

Where $E(R_t) = \mu$ , $Var(R_t) = \sigma^2$

Where $y_t = C + \sum_{i=1}^{k} a_i y_{t-i} + \epsilon_t$ 3.46

$\epsilon_t = v_t\sqrt{h_t}$ 3.47

$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i}$ 3.48

Where $v_t$ is white noise term, then $\epsilon_t$ is normal distribution with $h_t$ as variance and zero mean, therefore

$$p(\epsilon_t / \epsilon_{t-1}, \dots \epsilon_0) = \frac{1}{\sqrt{2\pi h_t}} e^{-\frac{\epsilon_t^2}{2h_t}} \qquad 3.49$$

The log- likelihood function of parameter vector $\theta = (\alpha_0, \alpha_1 \dots \alpha_q, \beta_1, \dots, \beta_p)^T$ is

$$L(\theta) = \sum_{i=q+1}^{n} L_t(\theta) = \sum_{t=q+1}^{n}\{-\frac{1}{2} In 2\pi - \frac{1}{2} In\sigma h_t - \frac{\epsilon_t^2}{2h_t}\} \qquad 3.50$$

Differentiating with respect to $\theta$ we have

$$\frac{\partial L_t(\theta)}{\partial\theta} = \left(\frac{\epsilon_t^2}{2h_t^2} - \frac{1}{2h_t}\right)\frac{\partial h_t}{\partial\theta}$$

With second derivative as

$$\frac{\partial^2 L_t(\theta)}{\partial\theta\partial\theta^T} = \left(\frac{\epsilon_t^2}{2h_t^2} - \frac{1}{2h_t}\right)\frac{\partial^2 h_t}{\partial\theta\partial\theta^T} + \left(\frac{1}{2h_t^2} - \frac{\epsilon_t^2}{2h_t^3}\right)\frac{\partial h_t}{\partial\theta}\frac{\partial h_t}{\partial\theta^T} \qquad 3.51$$

$$\frac{\partial h_t}{\partial\theta} = \left(1,\ \epsilon_{t-1}^2, \dots, \epsilon_{t-q}^2,\ \mathrm{h}_{t-1}, \dots, \mathrm{h}_{t-p}\right) + \sum_{i=1}^{p}\beta_i\frac{\partial h_{t-i}}{\partial\theta}$$

Thus the gradient is

$$\nabla L(\theta) = \frac{1}{2}\sum_{t=q+1}^{n}\left(\frac{\epsilon_t^2}{h_t^2} - \frac{1}{h_t}\right)\frac{\partial h_t}{\partial\theta} \qquad 3.52$$

And Fisher Information matrix is

$$J = \sum_{t=q+1}^{n} E\left[\left(\frac{\epsilon_t^2}{h_t^2} - \frac{1}{h_t}\right)\frac{\partial^2 h_t}{\partial\theta\partial\theta^T} + \left(\frac{1}{2h_t^2} - \frac{\epsilon_t^2}{h_t^3}\right)\frac{\partial h_t}{\partial\theta}\frac{\partial h_t}{\partial\theta^T}\right]$$

$$= -\frac{1}{2}\sum_{t=q+1}^{n} E\left(\frac{1}{h_t^2}\frac{\partial h_t}{\partial\theta}\frac{\partial h_t}{\partial\theta^T}\right) \qquad 3.53$$

## 3.8 The Generalised Length Biased Scaled –t GARCH (1,1) Model

The generalised length biased scaled -t is derived from Fisher concept of weighted function using student-t as proper probability density function *f(y)*. TheStudent –t distribution is given by

$$f(y)=\frac{\Gamma\left(\frac{v+1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)\sqrt{\pi(v-2)\sigma^2}}\left[1+\frac{(y-\mu)^2}{(v-2)\sigma^2}\right]^{-\left(\frac{v+1}{2}\right)} \qquad 3.54$$

Fisher (1934) introduce the concept of weighted distribution, Rao (1965) develop this concept in general term by in connection with modelling statistical data where the usual practice of using standard distribution were not found to be appropriate. Cox (1962) introduce statistical interpretation of length biased

Let $(\delta,\xi,P)$ be a probability space, $Y:\delta\rightarrow H$ be a random variable where H = ( a,b) be an interval on real line with 0< a, b with a< b can be finite or infinite. When the distribution function f(y) of Y is absolutely continuous (discrete) with probability density function and w(y) be a non –negative weighted function satisfying

$\mu_w=E\left(w(y)\right)<\infty$ then the random variables of $Y_w$ having pdf

$$f_w(y)=\frac{w(y)f(y)}{E\left(w(y)\right)}, a<y<b \qquad 3.55$$

Where $E\left(w(y)\right)=\int_{-\infty}^{\infty} w(y)f(y)dy$ , $-\infty<y<\infty$ is said to have weighted distribution

The length biased distribution is obtainedwhen the weighted function relied on the length of units of interest  (i.e.  w(y) = y) . The pdf of a length biased random variable is defined as :

$$g(y) = \frac{yf(y)}{\mu} \qquad 3.56$$

y is the the length of unit of interest w(y), f(y) is the baseline distribution, μ scaled parameter and g(y) is the probability density function. In this work, our based line distribution is Student-t distribution, which is stated in equation (3.56).

Substitute (3.54) into (3.56), we have:

$$g(y) = \frac{\Big)\overline{\frac{v+1}{2}}}{\mu \Big)\overline{\frac{v}{2}}\sqrt{\pi(v-2)\sigma^2}}\, y\left[1+\frac{(y-\mu)^2}{(v-2)\sigma^2}\right]^{-\left(\frac{v+1}{2}\right)} \qquad 3.57$$

The likelihood function of (3.57) is :

$$\prod_{i=1}^{n} g(y) = \frac{\left(\Big)\overline{\frac{v+1}{2}}\right)^n}{\mu^n\left(\Big)\overline{\frac{v}{2}}\right)^n\left(\pi(v-2)\sigma^2\right)^{\frac{n}{2}}}\prod_{i=1}^{n} y_i \prod_{i=1}^{n}\left[1+\frac{(y-\mu)^2}{(v-2)\sigma^2}\right]^{-\left(\frac{v+1}{2}\right)} \qquad 3.58$$

Taking the log likelihood of (3.58) we have:

$$L = \log\prod_{i=1}^{n} g(y) = n\log\Big)\overline{\frac{v+1}{2}} - n\log\mu - n\log\Big)\overline{\frac{v}{2}} - \frac{n}{2}\log\left[\pi(v-2)\sigma^2\right] + \sum_{i=1}^{n}\log y_i - \left(\frac{v+1}{2}\right)\sum_{i=1}^{n}\log\left[1+\frac{(y-\mu)^2}{(v-2)\sigma^2}\right] \qquad 3.59$$

Considering GARCH(1,1) with AR(1) specification in equation 3.61 and 3.60, respectively.

$$y_t = c + d_1 y_{t-1} + \varepsilon_t \qquad 3.60$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \qquad 3.61$$

If $\varepsilon_t$ is Generalised length biased scaled –t distribution we have

$$P\left(\varepsilon_t / \varepsilon_{t-1}, \dots, \varepsilon_0\right) = \frac{\Gamma\left(\frac{v+1}{2}\right) y_i \left[1 + \frac{\varepsilon_t^2}{(v-2)\sigma^2}\right]^{-\left(\frac{v+1}{2}\right)}}{\mu \Gamma\left(\frac{v}{2}\right)\sqrt{\pi(v-2)\sigma_t^2}} \qquad 3.62$$

The loglikelihood is

$$L = n\log\Gamma\frac{v+1}{2} - n\log\mu - n\log\Gamma\frac{v}{2} - \frac{n}{2}\log\left[\pi(v-2)\sigma^2\right] + \sum_{i=1}^{n}\log y_i - \left(\frac{v+1}{2}\right)\sum_{i=1}^{n}\log\left[1 + \frac{\varepsilon_t^2}{(v-2)\sigma_t^2}\right]$$

$$L = n\log\Gamma\frac{v+1}{2} - n\log\left[c + a_1 y_{t-1}\right] - n\log\Gamma\frac{v}{2} - \frac{n}{2}\log\left[\pi(v-2)\sigma_t^2\right] + \sum_{i=1}^{n}\log\left[c + \alpha_1 y_{t-1} + \varepsilon_t\right] - \left(\frac{v+1}{2}\right)\sum_{i=1}^{n}\log\left[1 + \frac{\varepsilon_t^2}{(v-2)\left[\alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2\right]}\right]$$

$$L(\theta) = n\log\frac{\Gamma\frac{v+1}{2}}{\sqrt{\pi(v-2)}\,\Gamma\frac{v}{2}} - \frac{1}{2}\sum_{t=1}^{n}\log\sigma_t^2 - \left(\frac{v+1}{2}\right)\sum_{t=1}^{n}\left[\log\left(1 + \frac{\varepsilon_t^2}{\sigma_t^2(v-2)}\right)\right] + \sum\log\left[\alpha_0 + \alpha_1 y_{t-1} + \varepsilon_t\right] - n\log\mu \qquad 3.63$$

$$l(\theta) = \left(\alpha_{0,}\alpha_{1,}\beta_1\right)^T$$

If we let $h_t = \sigma_t^2$ then

$$L(\theta)=n\log\frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{\pi(v-2)}\,\Gamma\left(\frac{v}{2}\right)}-\frac{1}{2}\sum_{t=1}^{n}\log h_t-\left(\frac{v+1}{2}\right)+\sum_{t=1}^{n}\left[\log\left(1+\frac{\varepsilon_t^2}{h_t(v-2)}\right)\right]+\sum\log\left[\alpha_0+\alpha_1 y_{t-1}+\varepsilon_t\right] \qquad 3.64$$

Differentiating this with respect to $\theta$ we have

$$\frac{dL(\theta)}{d\theta}=\left[\sum_{t=1}^{n}\frac{\frac{2\varepsilon_t^2}{h_t(v-2)}}{\left[\left(1+\frac{\varepsilon_t^2}{h_t(v-2)}\right)\right]}+\sum_{t=1}^{n}\frac{1}{\alpha_0+\alpha_1 y_{t-1}+\varepsilon_t}\right]\frac{dh_t}{d\theta}$$

$$\frac{dL(\theta)}{d\theta}=\left[\sum_{t=1}^{n}\frac{2\varepsilon_t^2}{h_t(v-2)+\varepsilon_t^2}+\sum_{t=1}^{n}\left(\frac{1}{\alpha_0+\alpha_1 y_{t-1}+\varepsilon_t}\right)\right]\frac{dh_t}{d\theta} \qquad 3.65$$

$\theta=\left(\alpha_{0,}\alpha_{1,}\beta_1\right)^T$ and $h_t=\sigma_t^2=\alpha_0+\alpha_1\varepsilon_{t-1}^2+\beta_1\sigma_{t-1}^2$

$$\frac{dh_t}{d\alpha_0}=1 \qquad 3.\ 66$$

$$\frac{dh_t}{d\alpha_1}=\varepsilon_{t-1}^2 \qquad 3.67$$

$$\frac{dh_t}{d\beta_1}=\sigma_{t-1}^2 \qquad 3.68$$

## 3.9 Limitation of GARCH (p, q) Model

1. GARCH Model fail to model leverage effect, it does not completely capture heavy tail characteristic of high frequency financial time series.

2. There are restrictions i.e. $\alpha_0 > 0$ , $\alpha_i \geq 0$ and $\beta_j \geq 0$ for conditional variance to be positive so therefore GARCH (p,q) models fails to respond or capture asymmetric effect properly.

These limitations lead to various use and invention of extensions of nonlinear GARCH models. In this work Asymmetric Power ARCH (p,q) model would be consider because its response to leverage effect and recent use in trade market. The model is characterise as one the best extension of non-linear GARCH model.

## 3.10 APARCH(p,q) Model Specification With Generalised Beta- Skewed t (GBST) Innovation

The Asymmetric Power ARCH (APARCH) model by Ding, Engle and Granger (1993) can be express as: $r_t = \mu + \varepsilon_t \quad , \quad \varepsilon_t = \sigma_t z_t \quad , \quad z_t \sim N(0,1)$ 3.69

$$\sigma_t^{\delta} = \alpha_0 + \sum_{i=1}^{q} \alpha_i \left(|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i}\right)^{\delta} + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^{\delta} \qquad 3.70$$

Where

$$\alpha_0 > 0, \delta \geq 0, \alpha_i \geq 0, i = 1,...q, -1 < \gamma_i < 1, i = 1,...q, \beta_j \geq 0, j = 1,...p$$

The $\gamma_i$ parameter permit us to catch the asymmetric effects. The conditional standard deviation process and the asymmetric absolute residuals in the model were imposed in term of a Box- transformation. The well-known Leverage effect is the asymmetric response of volatility to negative and positive shocks.

Consider the mean equation to be:

**AR (1)**

$$y_t = \phi_1 y_{t-1} + \varepsilon_t \qquad 3.71$$

Then, McDonald (1984) introduced the first kind of generalised beta distribution link function, Jones (2008) advocates its tractability, which can be characterised by the following link function:

$$g(y) = \frac{c}{\beta(a,b)} F(x)^{ac-1} \left[1 - F(x)^{c}\right]^{b-1} f(x) \qquad 3.72$$

Where a, b and c are shape parameters, f(x) and F(x) are density and cumulative functions respectively.

Let Student – t distribution be given as

$$f(x)=\frac{\Gamma\left(\frac{v+1}{2}\right)}{\sigma\,\Gamma\left(\frac{v}{2}\right)\sqrt{\pi(v-2)}}\left[1+\left(\frac{y-\mu}{\sigma}\right)^{2}\frac{1}{v-2}\right]^{-\frac{v+1}{2}} \qquad 3.73$$

Where $F(x)=I_{x(abc)}$ is an incomplete beta function. 3.74

if we put (75) and (76) in (74) we have the GBST distribution

$$g(y_t)=\frac{c}{\beta(a,b)}I^{ac-1}\left[1-I^{c}\right]^{b-1}\frac{\Gamma\left(\frac{v+1}{2}\right)}{\sigma\,\Gamma\left(\frac{v}{2}\right)\sqrt{\pi(v-2)}}\left[1+\left(\frac{y-\mu}{\sigma}\right)^{2}\frac{1}{v-2}\right]^{-\frac{v+1}{2}} \qquad 3.75$$

If we assume that $\varepsilon_t \sim (v,\mu,a,b,c)$

We have

$$g(y_t)=\frac{c}{\beta(a,b)}I^{ac-1}\left[1-I^{c}\right]^{b-1}\frac{\Gamma\left(\frac{v+1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)\sqrt{\pi(v-2)\sigma_t^{2}}}\left[1+\frac{\varepsilon_t^{2}}{\sigma_t^{2}}\frac{1}{v-2}\right]^{-\frac{v+1}{2}} \qquad 3.76$$

The log likelihood is

$$\prod_{t=2}^{n} g(y_t) = L = \frac{c^n}{\beta(a,b)^n} \prod_{t=2}^{n} I^{ac-1} \Gamma\left(\frac{v+1}{2}\right)^n \frac{\prod_{t=2}^{n}\left[1-I^c\right]^{b-1}}{\left[\pi(v-2)\sigma_t^2\right]^{\frac{n}{2}}} \frac{1}{\Gamma\left(\frac{v}{2}\right)^n} \prod_{t=2}^{n}\left[1+\frac{\varepsilon_t^2}{\sigma_t^2(v-2)}\right]^{-\frac{v+1}{2}} \qquad 3.77$$

Taking the log of (67) we have

$$\begin{aligned} l = LogL &= n\log c - n\log B(a,b) + n\log\Gamma\frac{v+1}{2} - n\log\Gamma\frac{v}{2} - \frac{n}{2}\log\left[\pi(v-2)\sigma_t^2\right] \\ &+(ac-1)\sum_{t=2}^{n} LogI + (b-1)\sum_{t=2}^{n}\log(1-I^c) - \left(\frac{v+1}{2}\right)\sum_{t=2}^{n}\log\left[1+\frac{\varepsilon_t^2}{\sigma_t^2(v-2)}\right] \end{aligned} \qquad 3.78$$

With AR (1) –APARCH (1,1), we substitute for $\sigma_t^2$ and $\varepsilon_t^2$ in (59) and (61) into (68) we have

$$\begin{aligned} l &= n\log c - n\left[\log\Gamma a + \log\Gamma b - \log\Gamma a+b\right] + n\log\Gamma\frac{v+1}{2} - n\log\Gamma\frac{v}{2} - \frac{n}{2}\left[\begin{array}{l}\log\pi + \log(v-2) + \\ \sum_{t=2}^{n}\log\left[\alpha_0 + \alpha_1\left(\left|\varepsilon_{t-1}\right| - \gamma_1\varepsilon_{t-1}\right)^2\right]\end{array}\right] + \\ &(ac-1)\sum_{t=2}^{n} LogI + (b-1)\sum_{t=2}^{n}\log(1-I^c) - \left(\frac{v+1}{2}\right)\sum_{t=2}^{n}\log\left[1+\frac{\left(Y_t - \phi_1 Y_{t-1}\right)^2}{\alpha_0 + \alpha_1\left(\left|\varepsilon_{t-1}\right| - \gamma_1\varepsilon_{t-1}\right)^2(v-2)}\right] \end{aligned} \qquad 3.79$$

Differentiate $l(\theta)$ and equate to zero with rest to $\theta^T = (a,b,c,\alpha_0,\alpha_1,\gamma_1,\phi_1)$ we have

$$\frac{dl}{da} = -n\frac{n\Gamma^{1}a}{\Gamma a} + n\frac{\Gamma^{1}a+b}{\Gamma a+b} + c\sum_{t=2}^{n}\log I \qquad 3.80$$

$$\frac{dl}{db} = -n\frac{n\Gamma^{1}b}{\Gamma b} + n\frac{\Gamma^{1}a+b}{\Gamma a+b} + c\sum_{t=2}^{n}\log(1-I^c) \qquad 3.81$$

$$\frac{dl}{dc} = \frac{n}{c} + a\sum_{t=2}^{n}\log I + (b-1)\sum_{t=2}^{n}\frac{\frac{\delta}{\delta c}I^c}{(1-I^c)} \qquad 3.82$$

$$\frac{dl}{d\alpha_0} = \sum_{t=2}^{n} \frac{1}{\alpha_0 + \alpha_1 \left( \left| \varepsilon_{t-1} \right| - \gamma_1 \varepsilon_{t-1} \right)^2} \qquad 3.83$$

$$\frac{dl}{d\alpha_1} = \sum_{t=2}^{n} \frac{\left( \left| \varepsilon_{t-1} \right| - \gamma_1 \varepsilon_{t-1} \right)^2}{\alpha_0 + \alpha_1 \left( \left| \varepsilon_{t-1} \right| - \gamma_1 \varepsilon_{t-1} \right)^2} \qquad 3.84$$

$$\frac{dl}{d\gamma_1} = -2\alpha_1 \sum_{t=2}^{n} \frac{\varepsilon_{t-1}}{\alpha_0 + \alpha_1 \left( \left| \varepsilon_{t-1} \right| - \gamma_1 \varepsilon_{t-1} \right)^2} \qquad 3.85$$

$$\frac{dl}{d\phi_1} = (v+1) \sum_{t=2}^{n} \frac{Y_{t-1} \left( Y_t - \phi_1 Y_{t-1} \right)}{\left[ 1 + \frac{\left( Y_t - \phi_1 Y_{t-1} \right)^2}{\alpha_0 + \alpha_1 \left( \left| \varepsilon_{t-1} \right| - \gamma_1 \varepsilon_{t-1} \right)^2 (v-2)} \right]} \qquad 3.86$$

## 3.11 OTHER PARAMETER ESTIMATIONS IN THE LITERATURE: NORMAL, STUDENT-T AND GED INNOVATIONS

In the literature the most recent error innovation used along with volatility models are Normal, Student-t and GED. Below are parameter estimations of the three innovation: see Yaya *et al*, (2013) ,for Normal distribution ,the Log-likelihood is

$$\boldsymbol{l_t} = -\frac{\mathbf{1}}{\mathbf{2}} \left[ N log(2\pi) + \sum_{i=1}^{N} \frac{\varepsilon_t^2}{\sigma_t^2} + \sum_{i=1}^{N} log\sigma_t^2 \right] \qquad 3.87$$

$\boldsymbol{\varepsilon_t} = \boldsymbol{z_t \sigma_t}$where $z_t = \frac{\varepsilon_t}{\sigma_t}$

Equation 98 is the log-likelihood $\boldsymbol{l_t}$of Normal in GARCH models, N is the sample sizes of the series, $\varepsilon_t$ is the white noise ,$z_t is$ sequence of identical independent random variables and $\sigma_t^2$ *is the* conditional variance.

The Log-likelihood for Student-t distribution is

$$l_t = -\frac{1}{2}\left[N \log\left(\frac{\pi(v-2)\gamma\left(\frac{v}{2}\right)^2}{\gamma\left(\frac{v+1}{2}\right)^2}\right) + \sum_{i=1}^{N} \log \sigma_t^2 + (v-1)\sum_{i=1}^{N} log\left(1 + \frac{\epsilon_t^2}{\sigma_t^2(v-1)}\right)\right] \qquad 3.88$$

In the estimation in equation (3.89) v is the degree of freedom and $\gamma(.)$ Is the gamma function, for GED it is

$$l_t = -\frac{1}{2}\left[N \log\left(\frac{\gamma\left(v^{-1}\right)^3}{\gamma(3v^{-1})\,\gamma\left(\frac{v}{2}\right)^2}\right) + \sum_{i=1}^{N} \log \sigma_t^2 + (v-1)\sum_{i=1}^{N} log\left(\frac{\gamma\left(3v^{-1}\right)\epsilon_t^2}{\sigma_t^2(v^{-1})}\right)\right] \qquad 3.89$$

While v in equation (3.89) is the thickness of tail parameter. The log likelihood functions in 3.63, 3.79, 3.87,3.88 and 3.89 are simplified using R code.

# Chapter Four

# RESULTS AND DISCUSSION

## 4.1 Preamble

In this section, an extensive empirical analysis of conditional variance and mean of returns of Nigeria Stock Exchange (NSE) is carried out on GARCH models with various error innovations. The forecast performance of the existing ones are compared with the newly proposed error innovations of GARCH models. Results and interpretation were discussed extensively. Transformation or logarithms of the assets price that is, the returns were obtained in order to capture stylised facts of the volatility more clearly.

## 4.2 Results And Discussion

Returns series is obtained by log transformation of assets prices given as

$$R_t = logY_t - logY_{t-1} = \log\left(\frac{Y_t}{Y_{t-1}}\right) = log\left(1 + \frac{Y_t - Y_{t-1}}{Y_{t-1}}\right) \qquad 4.1$$

Let All Share index price be $Y_t$ for day t. The Autocorrelation and Partial functions plot in figure 4.1 and 4.2 respectively point towards AR(1) model as a model to determine the estimate of returns series. Table 4.1 shows the descriptive statistics for returns of Nigeria Stock Exchange (NSE) index between January 2000 to December 2015, the existence of negative skewness and playkurtosis indicate anomalous in the returns series. The Time plot, which is the first step to examine hidden characteristic reveals non-stationarity, patterns and clustered volatility. Figures 4.3 is plot for returns series when series have undergo transformation or attain stationary while figure 4.4 displayed the -time plot when the series is non stationary that is original plot of series.

**Table 4. 1 Descriptive Statistics of Stock Returns**

| Index | Min | 1st Qu | Median | 3rd Qu | Mean | Max | SD | Skewness | Kurtosis | Shapiro – Wilk test |
|---|---|---|---|---|---|---|---|---|---|---|
| NSE | 1.00 | 48.75 | 96.50 | 144.20 | 96.40 | 191.00 | 55.43 | -0.001 | -1.226 | 0.954 |

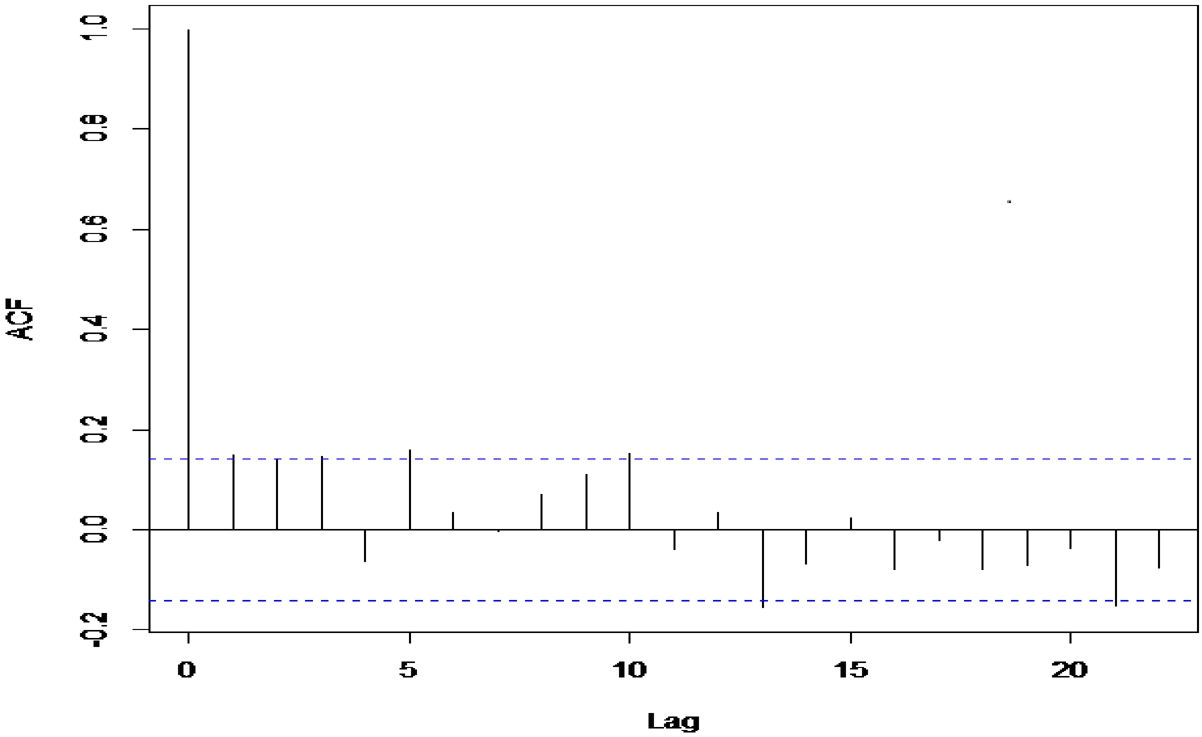


**Figure 4.1: Plot of Autocorrelation Function of Stock Returns.**

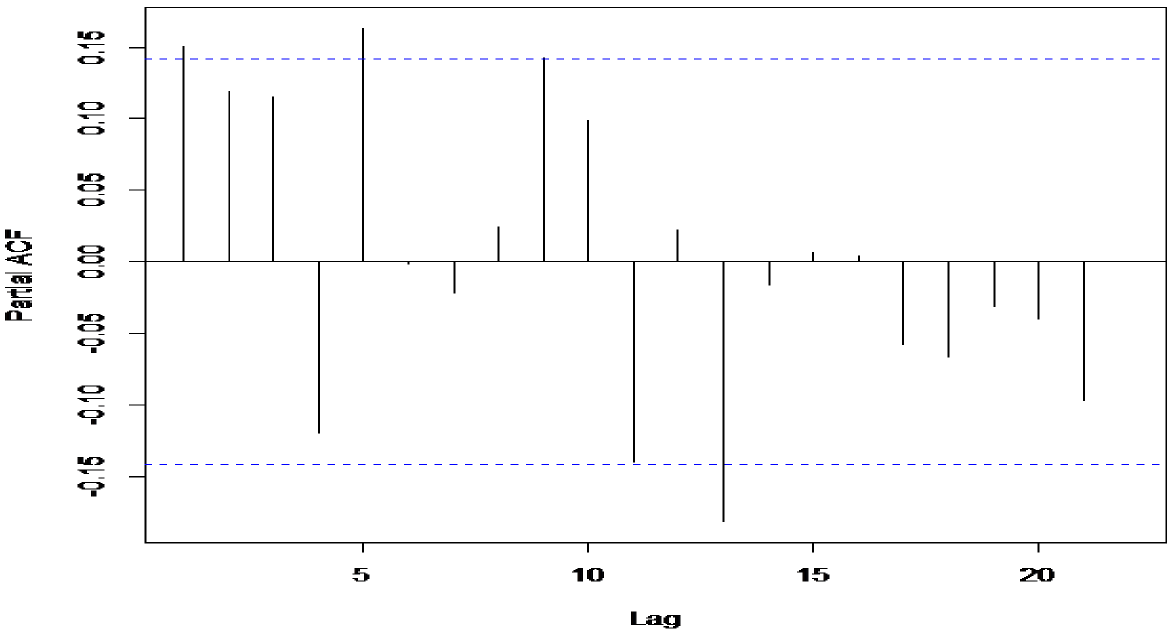


**Figure 4.2 Plot of Partial Autocorrelation Function of Stock Returns**

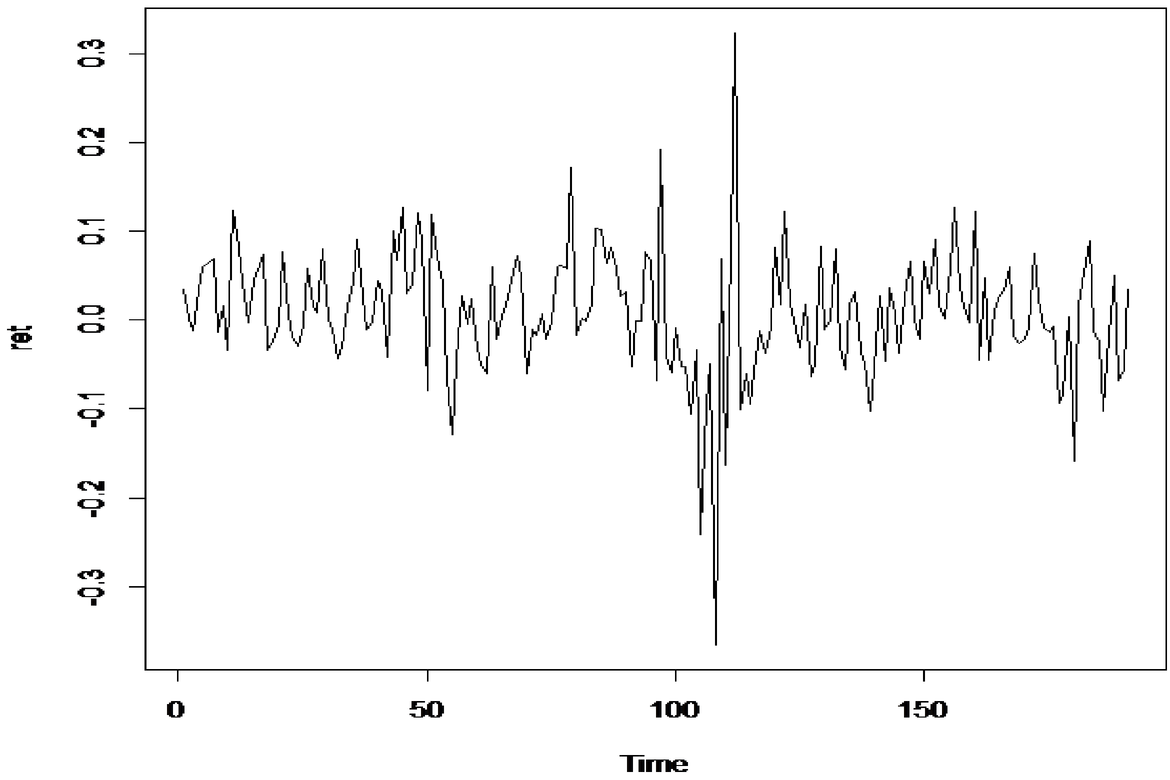


**Figure 4.3 Time Plot of Returns Series**

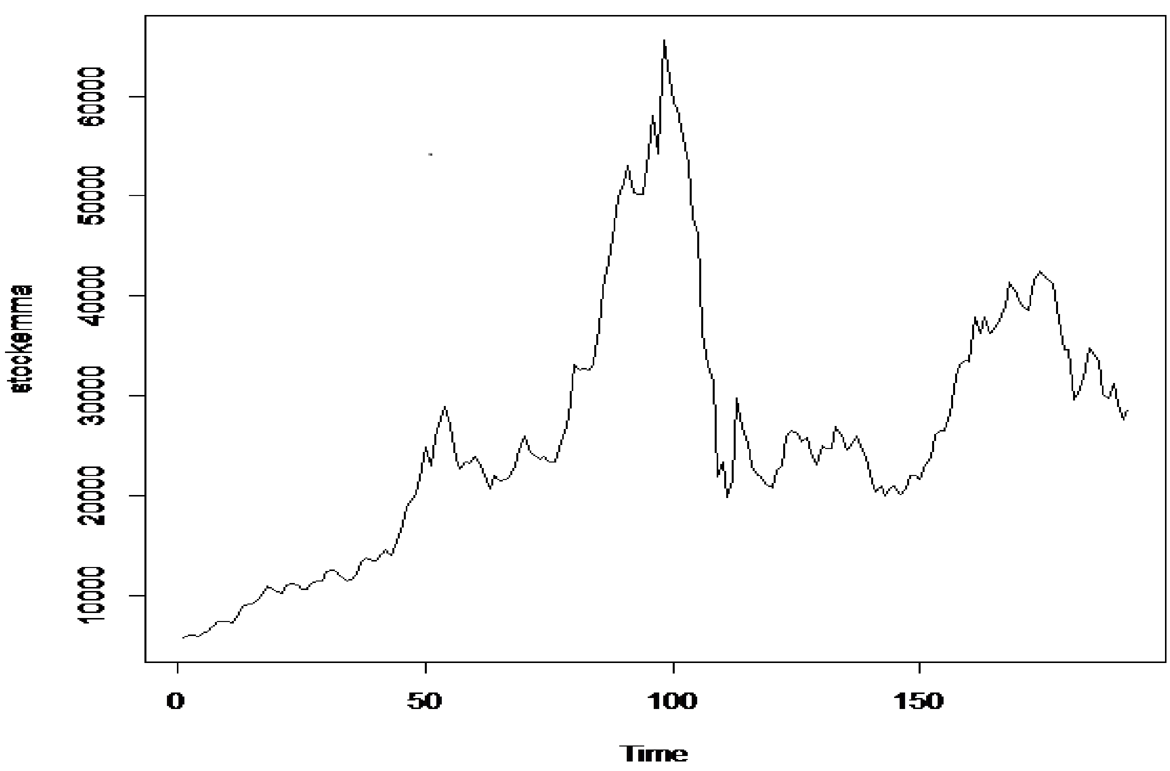


**Figure 4.4 Time plot of the Stock Series**

Table 4.2– 4.12 shows the Parameter estimate, Standard errors, t values and P value of existing and proposed models, they are analyse with the NSE data using R software. The estimation is divided into two, first is the mean equation which is Autoregressive AR (1) model. This was obtained from the autocorrelation and partial autocorrelation functions. The second part is the variance equation, which can specified by ARCH/GARCH models. In this study we considered GARCH (1, 1) model and APARCH (1, 1) model as the variance which can capture both symmetric and asymmetric underlying in financial series.. The models are estimated by method of Quasi Maximum likelihood Estimator, assuming the error innovations as normal, student –t, GED, Generalised Length Biased Scaled-t and Generalised Beta Skewed-t innovation. R statistical software was used to analyse the work.

**Table 4.2 Parameter Estimation for GARCH – Normal**

| | **Estimate** | **Std. Error** | **t- Ratio** | **P value** |
|---|---|---|---|---|
| $\boldsymbol{\mu}$ | 82.075 | 4.490 | 18.280 | < 2e-16 *** |
| $\alpha_0$ | 160.753 | 80.396 | 2.000 | 0.0456 * |
| $\alpha_1$ | 0.792 | 0.177 | 4.466 | 7.96e-06 *** |
| $\beta_1$ | 0.139 | 0.166 | 0.836 | 0.4030 |

Significant. levels: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

From table 4.2, the location parameter $\mu$ , the ambient volatility parameter $\alpha_0$ and adjustment to past shock parameter $\alpha_1$ are highly significant as their P- values using significance level ($\alpha = 0.05$) while adjustment to past volatility parameter $\beta_1$ is not significant at 5%.

**Table 4.3 Parameter Estimation for GARCH –GED**

| | Estimate | Std. Error | T Ratio | P value |
|---|---|---|---|---|
| $\boldsymbol{\mu}$ | 69.015 | 1.131 | 61.033 | < 2e-16 *** |
| $\boldsymbol{\alpha_0}$ | 260.535 | 62.464 | 4.171 | 3.03e-05 *** |
| $\boldsymbol{\alpha_1}$ | 0.420 | 0.050 | 8.404 | < 2e-16 *** |
| $\boldsymbol{\beta_1}$ | 0.169 | 0.095 | 1.782 | 0.0747 |
| **V** | 10.000 | 1.636 | 6.114 | 9.70e-10 *** |

Significant. levels: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

From table 4.3, the location parameter$\mu$ , the ambient volatility parameter $\alpha_0$, adjustment to past shock parameter $\alpha_1$ and degree of freedom parameter V are highly significant as their P- values using significance level ($\alpha = 0.05$) while Adjustment to past volatility parameter $\beta_1$is not significant at 5%.

**Table 4.4 Parameter Estimation for GARCH – Student.t**

| | **Estimate** | **Std. Error** | **t Ratio** | **P value** |
|---|---|---|---|---|
| $\boldsymbol{\mu}$ | 85.415 | 4.664 | 18.313 | < 2e-16 *** |
| $\boldsymbol{\alpha_0}$ | 153.872 | 87.196 | 1.765 | 0.077619 |
| $\boldsymbol{\alpha_1}$ | 1.000 | 0.224 | 4.463 | 8.09e-06 *** |
| $\boldsymbol{\beta_1}$ | 0.129 | 0.148 | 0.865 | 0.386799 |
| **V** | 10.000 | 2.697 | 3.708 | 0.000209 *** |

Significant. levels: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

From table 4.4, the location parameter $\mu$ , adjustment to past shock parameter $\alpha_1$ and degree of freedom parameter V are highly significant as their P- values using significance level ($\alpha \quad = 0.05$) while the ambient volatility parameter $\alpha_0$, adjustment to past volatility parameter $\beta_1$are not significant at 5%.

**Table 4.5 Parameter Estimation for GARCH - Beta Skewed-t**

| | Estimate | Std Error | t- Ratio | P value |
|---|---|---|---|---|
| $\mu$ | 0.870 | 0.001 | 870 | < 2e-16 *** |
| $\alpha_0$ | 4.785 | 0.001 | 2658.333 | 0.077619 |
| $\alpha_1$ | 0.301 | 0.0002 | 150.5 | 8.09e-06 *** |
| $\beta_1$ | 0.187 | 0.001 | 187 | 0.386799 |
| **V** | 9.365 | 0.001 | 9365 | 0.000209 *** |
| $\xi$ | 0.440 | 0.0002 | 2200 | < 2e-16 *** |

Significant. levels: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

From table 4.5, the location parameter$\mu$ , adjustment to past shock parameter $\alpha_1$ shape parameter $\boldsymbol{\xi}$ and degree of freedom parameter V are highly significant as their P- values using significance level ( $\alpha \quad = 0.05$ ) while the ambient volatility parameter $\alpha_0$ , adjustment to past volatility parameter $\beta_1$are not significant at 5%.

**Table 4.6 Parameter Estimation for GARCH – Length Baised Scale-t**

| | **Estimate** | **Std. Error** | t – Ratio | **P value** |
|---|---|---|---|---|
| $\mu$ | 1.000 | 0.072 | 210.8044 | 0.00000 |
| $\alpha_0$ | 14.893 | 8.616 | 1.1905 | 0.09691 |
| $\alpha_1$ | 0.000 | 0.010 | 0.0000 | 1.00000 |
| $\beta_1$ | 0.988 | 0.012 | 81.1138 | 0.000011 |
| **V** | 2.124 | 0.026 | 125.1861 | 0.00000 |

Significant. levels: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

From table 4.6, the location parameter $\mu$ , adjustment to past volatility parameter $\beta_1$ and degree of freedom parameter V are highly significant as their P- values using significance level ($\alpha = 0.05$) while the ambient volatility parameter $\alpha_0$ adjustment to past shock parameter $\alpha_1$ are not significant at 5%.

**Table 4.7 Parameter Estimation for APARCH – Normal**

| | **Estimate** | **Std. Error** | **t – Ratio** | **P value** |
|---|---|---|---|---|
| $\boldsymbol{\mu}$ | 82.080 | 6.427 | 12.771 | < 2e-16 *** |
| $\boldsymbol{\alpha_0}$ | 160.090 | 455.328 | 0.352 | 0.725143 |
| $\boldsymbol{\alpha_1}$ | 0.765 | 0.200 | 3.821 | 0.000133 *** |
| $\boldsymbol{\beta_1}$ | 0.151 | 0.179 | 0.845 | 0.397948 |
| $\boldsymbol{\gamma_1}$ | -0.042 | 0.065 | -0.646 | 0.518469 |
| $\boldsymbol{\delta}$ | 0.151 | 2.382 | 0.840 | 0.401172 |

Significant. levels: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

From table 4.7, the location parameter $\mu$ and adjustment to past shock parameter $\alpha_1$ are highly significant as their P- values at significance level ($\alpha = 0.05$) while the ambient volatility parameter $\alpha_0$, adjustment to past volatility parameter $\beta_1$ and asymmetric parameters

$\gamma_1$ and $\delta$ are not significant at 5%.

**Table 4.8 Parameter Estimation for APARCH GED**

| | **Estimate** | **Std. Error** | **t – Ratio** | **P value** |
|---|---|---|---|---|
| $\boldsymbol{\mu}$ | 69.558 | 1.253 | 55.532 | < 2e-16 *** |
| $\boldsymbol{\alpha_0}$ | 16.455 | 6.366 | 2.585 | 0.00974 ** |
| $\boldsymbol{\alpha_1}$ | 0.359 | 0.044 | 8.076 | 6.66e-16 *** |
| $\boldsymbol{\beta_1}$ | 0.291 | 0.089 | 3.258 | 0.00112 ** |
| $\boldsymbol{\gamma_1}$ | -0.120 | 0.063 | -1.901 | 0.05728 |
| $\boldsymbol{\delta}$ | 1.166 | 0.446 | 2.616 | 0.00889 ** |
| **V** | 10.000 | 1.895 | 5.277 | 1.31e-07 *** |

Significant. levels: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

From table 4.8, the location parameter $\mu$, the ambient volatility parameter $\alpha_0$, adjustment to past volatility parameter $\beta_1$ ,adjustment to past shock parameter $\alpha_1$, asymmetric parameter $\delta$ and degree of freedom V are highly significant as their P-values at significance level ($\alpha = 0.05$) while asymmetric parameter $\gamma_1$ is not significant at 5%.

**Table 4.9 Parameter Estimation for APARCH STD**

| | Estimate | Std. Error | t – Ratio | P value |
|---|---|---|---|---|
| $\boldsymbol{\mu}$ | 85.342 | 6.578 | 12.974 | < 2e-16 *** |
| $\boldsymbol{\alpha_0}$ | 156.794 | 338.295 | 0.463 | 0.643 |
| $\boldsymbol{\alpha_1}$ | 0.981 | 0.349 | 2.811 | 0.004943 ** |
| $\boldsymbol{\beta_1}$ | 0.133 | 0.158 | 0.846 | 0.398 |
| $\boldsymbol{\gamma_1}$ | -0.037 | 0.067 | -0.557 | 0.578 |
| $\boldsymbol{\delta}$ | 2.000 | 1.912 | 1.046 | 0.296 |
| **V** | 10.000 | 2.703 | 3.700 | 0.000216 *** |

Significant. levels: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

From table 4.9, the location parameter $\mu$, adjustment to past shock parameter $\alpha_1$, and degree of freedom V are highly significant as their P- values at significance level ($\alpha = 0.05$) while the ambient volatility parameter $\alpha_0$, adjustment to past volatility parameter $\beta_1$ asymmetric parameters $\gamma_1$ and $\delta$ are not significant at 5%.

**Table 4.10 Parameter Estimation for APARCH-Beta-Skew-t**

| | **Estimate** | **Std. Error** | **t – Ratio** | **P value** |
|---|---|---|---|---|
| $\boldsymbol{\mu}$ | 4.785 | 0.0018 | 870 | < 2e-16 *** |
| $\boldsymbol{\alpha_0}$ | 0.870 | 0.0010 | 2658.333 | 0.077619 |
| $\boldsymbol{\alpha_1}$ | 0.301 | 0.0002 | 150.5 | 8.09e-06 *** |
| $\boldsymbol{\beta_1}$ | 0.187 | 0.0010 | 187 | 0.386799 |
| **V** | 9.365 | 0.0010 | 9365 | 0.000209 *** |
| $\boldsymbol{\xi}$ | 0.440 | 0.0002 | 2200 | < 2e-16 *** |

Significant. levels: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

From table 4.10, the location parameter $\mu$ , shape parameter $\xi$, adjustment to past shock parameter $\alpha_1$, and degree of freedom V are highly significant as their P- values at significance level ($\alpha = 0.05$) while the ambient volatility parameter $\alpha_0$, adjustment to past volatility parameter $\beta_1$ are not significant at 5%.

**Table 4.11 Parameter Estimation for APARCH -Length Bias Scaled –t-**

| | **Estimate** | **Std. Error** | **t – Ratio** | **P Value** |
|---|---|---|---|---|
| $\boldsymbol{\mu}$ | 1.000 | 0.005 | 212.148 | 0.000 |
| $\boldsymbol{\alpha_0}$ | 14.829 | 8.662 | 1.711 | 0.087 |
| $\boldsymbol{\alpha_1}$ | 0.000 | 0.0100 | 0.000 | 1.000 |
| $\boldsymbol{\beta_1}$ | 0.988 | 0.012 | 82.120 | 0.000 |
| **V** | 2.100 | 0.017 | 125.186 | 0.000 |

Significant. levels: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

From table 4.11, the location parameter $\mu$, adjustment to past volatility parameter, $\beta_1$, and degree of freedom V are highly significant as their P- values at significance level ( $\alpha = 0.05$ ) while the ambient volatility parameter $\alpha_0$ *and* adjustment to past shock parameter $\alpha_1$ are not significant at 5%.

Table 4.12 to 4.17 displayed the model comparisons of real data on Nigeria Stock Exchange (NSE) monthly returns data between January 2000 to December 2015, monthly Central Bank of Nigeria Shortfall Excess Credit data between October 2005 to December 2014 and simulated data of size 50, 100, 500, 1000 were used to illustrate the models. The proposed models were compared with eight (8) existing GARCH models using Log- likelihood function, Root Mean Square Error (RMSE) Adjusted Mean Absolute Percentage Error (AMAPE), Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), Schwartz Criterion (SBIC), Hanna Quinn Information Criterion (HQIC) as performance evaluation tools.

**Table 4.12 Model Selection Criteria of NSE Index for Existing and Proposed Innovations**

| MODELS | Log likelihood | AIC | BIC | SBIC | HQIC | RANKING |
|---|---|---|---|---|---|---|
| GARCH | -995.69 | 10.413 | 10.481 | 10.412 | 10.441 | 7 |
| STD –GARCH | 1002.17 | 10.491 | 10.576 | 10.490 | 10.526 | 9 |
| GED –GARCH | -953.46 | 9.984 | 10.069 | 9.983 | 10.018 | 3 |
| GBST – GARCH | -1040.15 | 10.003 | 10.002 | 10.002 | 10.021 | 5 |
| GLBST – GARCH | -749.48 | 7.988 | 7.961 | 7.887 | 7.896 | 2 |
| APARCH | -995.48 | 10.432 | 10.534 | 10.430 | 10.473 | 8 |
| STD – APARCH | -1002.01 | 10.511 | 10.629 | 10.508 | 10.559 | 10 |
| GED – APARCH | -955.10 | 10.022 | 10.141 | 10.020 | 10.079 | 6 |
| GBST – APARCH | -1040.15 | 10.000 | 10.000 | 10.000 | 10.000 | 4 |
| GLBST-APARCH | -749.15 | 7.856 | 7.941 | 7.854 | 7.890 | 1 |

The model performance was determined how better the information was capture by Log-likelihood function and minimal values of the information criteria tools such as Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), Schwartz Criterion (SBIC) and Hanna Quinn Information Criterion (HQIC).From table XII considering the values of Log Likelihood and minimal model criteria values reveals that GLBST-APARCH (1,1) model (AIC = 7.856, BIC=7.941, SBIC= 7.854 and HQIC= 7.890), GLBST- GARCH(1, 1)model (AIC= 7.988,BIC=7.961, SBIC= 7.887 and HQIC= 7.896) and GED-GARCH(1, 1) model (AIC = 9.984,BIC=10.069, SBIC= 9.983and HQIC= 10.018) out performed normal error innovations of GARCH (1, 1) model (AIC = 10.413, BIC= 10.481, SBIC= 1-.412 and HHQIC= 10.441) and APARCH(1, 1) model (AIC = 10.432, BIC=10.534, SBIC= 10.430and HQIC= 10.473) respectively.

**TABLE 4.13 Model Selection Criteria for Excess Credit Data**

| MODELS | Log likelihood | AIC | BIC | SBIC | HQIC | RANKING |
|---|---|---|---|---|---|---|
| GARCH | -208.83 | 8.346 | 8.497 | 8.335 | 8.404 | 4 |
| STD –GARCH | -210.22 | 8.440 | 8.629 | 8.423 | 8.512 | 7 |
| GED –GARCH | -201.62 | 8.103 | 8.292 | 8.086 | 8.175 | 3 |
| GBST –GARCH | -222.09 | 9.213 | 9.173 | 9.182 | 9.180 | 10 |
| GLBST –GARCH | -749.15 | 6.698 | 6.697 | 6.677 | 6.906 | 2 |
| APARCH | -208.43 | 8.409 | 8.636 | 8.385 | 8.496 | 6 |
| STD –APARCH | -209.83 | 8.503 | 8.768 | 8.471 | 8.604 | 8 |
| GED –APARCH | -200.80 | 8.149 | 8.414 | 8.117 | 8.250 | 5 |
| GBST –APARCH | -223.01 | 9.031 | 9.013 | 9.015 | 9.023 | 9 |
| GLBST-APARCH | -417.72 | 6.269 | 6.372 | 6.266 | 6.310 | 1 |

Table 4.13 is the models performance on Shortfall/ Excess Credit Data Between October 2005 - December 2014, considering the values of Log Likelihood and minimal model criteria values , it reveals that GLBST- APARCH (1,1) model (AIC = 6.269, BIC=6.372, SBIC= 6.266 and HQIC= 6.310), GLBST- GARCH(1, 1)model (AIC= 6.698, BIC= 6.697, SBIC= 6.677 and HQIC= 6.906) and GED-GARCH(1, 1) model (AIC = 8.103, BIC= 8.292, SBIC= 8.086 and HQIC=8.175) out performed normal error innovations of GARCH (1, 1) model (AIC = 8.346, BIC=8.497 , SBIC=8.335 and HQIC=8.404) and APARCH(1, 1) model (AIC = 8.409, BIC = 8.636, SBIC= 8.385 and HQIC= 8.496) respectively.

## 4.3 Simulation

To validate the proposed innovations, simulation was carried out on sample sizes 50, 100, 500 and 1000. The pattern of performance remain the same for simulated data except for sample size (n = 1000) with GLBST- APARCH (1,1) model and GBST -GARCH (1,1) model performed below expectation

**TABLE 4.14 Simulation Results for Sample Size N=50**

| **MODELS** | **Log likelihood** | AIC | BIC | SBIC | HQIC | RANKING |
|---|---|---|---|---|---|---|
| GARCH | -67.116 | 2.8446 | 2.9976 | 2.8330 | 2.9029 | 6 |
| STD –GARCH | -68.568 | 2.9427 | 3.1339 | 2.9250 | 3.0155 | 8 |
| GED –GARCH | -62.479 | 2.6991 | 2.8903 | 2.6814 | 2.7719 | 2 |
| GBST–ARCH | -62.403 | 2.7687 | 2.9655 | 2.7655 | 2.8124 | 5 |
| GLBST–ARCH | -134.421 | 2.7684 | 2.8726 | 2.7654 | 2.8106 | 4 |
| APARCH | -68.130 | 2.9651 | 3.1946 | 2.9402 | 3.0525 | 9 |
| STD–APARCH | -74.363 | 3.2545 | 3.5220 | 3.2213 | 3.3564 | 10 |
| GED–PARCH | -66.078 | 2.9231 | 3.1908 | 2.8899 | 3.0250 | 7 |
| GBST–PARCH | -55.441 | 2.6606 | 2.8506 | 2.6593 | 2.8601 | 1 |
| GLBST-PARCH | 137.015 | 2.7650 | 2.8679 | 2.7621 | 2.8067 | 3 |

**TABLE 4.15 Simulation Results for Sample Size N=100**

| **MODELS** | **Log likelihood** | AIC | BIC | SBIC | HQIC | RANKING |
|---|---|---|---|---|---|---|
| GARCH | -157.913 | 3.2382 | 3.3424 | 3.2352 | 3.2804 | 4 |
| STD –GARCH | -158.975 | 3.2794 | 3.4097 | 3.2748 | 3.3322 | 7 |
| GED –GARCH | -157.789 | 3.2557 | 3.3860 | 3.2510 | 3.3084 | 6 |
| GBST–ARCH | -157.930 | 3.2548 | 3.4349 | 3.2452 | 3.2906 | 5 |
| GLBST–ARCH | -157.956 | 3.0000 | 3.0071 | 3.0071 | 3.0242 | 3 |
| APARCH | -172.240 | 3.5648 | 3.7211 | 3.5581 | 3.6280 | 9 |
| STD–APARCH | -165.590 | 3.4517 | 3.6341 | 3.4428 | 3.5255 | 8 |
| GED–PARCH | -587.347 | 11.8869 | 12.0693 | 11.8779 | 11.9607 | 10 |
| GBST–PARCH | -115.907 | 2.6814 | 2.8146 | 2.7832 | 2.8632 | 1 |
| GLBST-PARCH | -149.268 | 2.9194 | 3.0205 | 2.9166 | 2.9604 | 2 |

**TABLE 4.16 Simulation Results for Sample Size N=500**

| **MODELS** | **Log likelihood** | AIC | BIC | SBIC | HQIC | RANKING |
|---|---|---|---|---|---|---|
| GARCH | -240.294 | 2.842 | 3.1047 | 2.981 | 3.033 | 2 |
| STD –GARCH | -241.428 | 2.984 | 3.124 | 2.980 | 3.041 | 9 |
| GED –GARCH | -240.227 | 2.989 | 3.130 | 2.985 | 3.046 | 10 |
| GBST–ARCH | -235.633 | 2.965 | 3.030 | 2.943 | 2.968 | 8 |
| GLBST–ARCH | -240.344 | 2.956 | 3.036 | 2.955 | 2.989 | 3 |
| APARCH | -240.171 | 2.958 | 3.038 | 2.956 | 2.990 | 7 |
| STD–APARCH | -274.074 | 2.957 | 3.057 | 2.955 | 2.998 | 4 |
| GED–PARCH | -241.650 | 2.962 | 3.063 | 2.960 | 3.003 | 6 |
| GBST–PARCH | -261.778 | 2.952 | 2.960 | 2.951 | 2.950 | 5 |
| GLBST-PARCH | -276.591 | 2.738 | 2.823 | 2.7370 | 2.773 | 1 |

**TABLE 4.17 Simulation Results for Sample Size N=1000**

| **MODELS** | **Log likelihood** | AIC | BIC | SBIC | HQIC | RANKING |
|---|---|---|---|---|---|---|
| GARCH | -713.318 | 2.8692 | 2.9029 | 2.8691 | 2.8825 | 4 |
| STD –GARCH | -714.964 | 2.8798 | 2.9220 | 2.8796 | 2.8963 | 7 |
| GED –GARCH | -713.228 | 2.8729 | 2.9150 | 2.87271 | 2.8894 | 5 |
| GBST–ARCH | 706.182 | 2.8021 | 2.8993 | 2.8128 | 2.8817 | 1 |
| GLBST–ARCH | 713.310 | 2.8692 | 2.9030 | 2.8691 | 2.8825 | 3 |
| APARCH | -713.318 | 2.8772 | 2.9278 | 2.8769 | 2.8971 | 6 |
| STD-APARCH | -714.964 | 2.8878 | 2.9468 | 2.8874 | 2.9110 | 9 |
| GED–PARCH | -713.228 | 2.8809 | 2.9399 | 2.880528 | 2.9040 | 8 |
| GBST–PARCH | -668.265 | 2.8042 | 2.9222 | 2.8131 | 2.8820 | 2 |
| GLBST-PARCH | -708.171 | 2.9068 | 2.9411 | 2.9067 | 2.9203 | 10 |

**TABLE 4.18 Forecast Analysis Using Different Densities For The NSE Index**

| **MODELS** | **RMSE** | **AMAPE** | **RANKING** |
|---|---|---|---|
| GARCH | 0.757 | 0.757 | 10 |
| STD –GARCH | 0.389 | 0.342 | 6 |
| GED –GARCH | 0.626 | 0.684 | 9 |
| GBST–ARCH | 0.309 | **0.30**0 | **3** |
| GLBST–ARCH | 0.291 | 0.290 | **2** |
| APARCH | 0.538 | 0.894 | 8 |
| STD–APARCH | 0.350 | 0.342 | 5 |
| GED–PARCH | 0.457 | 0.480 | 7 |
| GBST–PARCH | 0.317 | 0.330 | 4 |
| GLBST-PARCH | 0.281 | 0.280 | **1** |

The forecast performance was obtained using Root Mean Square Error (RSME) and Adjusted Mean Absolute Percentage Error (AMAPE) as forecasting measures. The Forecast was reported by gradingvarious models in the studied with respect to minimal RSME and AMAPE for robust model for NSE index. The most suitable model is GLBST- APARCH (1, 1) model (RSME: 0.281 and AMAPE: 0.280), follow by GLBST –GARCH (1, 1) model (RMSE: 0.291, AMAPE: 0.291) and GBST – GARCH (1, 1) model (RMSE: 0.309, AMAPE: 0.300).

# Chapter Five

# SUMMARY, CONCLUSION AND CONTRIBUTION TO KNOWLEDGE

## 5.1 Preamble

In this section, summary of the results and most suitable models were discussed. Empirical features of financial time series were discussed based on NSE data; such as volatility clustering, serial correlation, thick tails existence due to non-normality of returns series, time series and returns plots, ACF and PACF plots and others

Selection criteria such as AIC, BIC, SQIC, HQIC and Forecast evaluation tools such as RSME and MSE were used to determined most suitable model and its forecast performance. It was concluded that GLBST -APARCH (1,1) model is the best model that explained NSE returns and it empirical characteristics. Further studies and contribution were also discussed.

## 5.2 Summary

This work has review the convectional GARCH (p, q) models, such as GARCH (1,1) model , APARCH (1,1) model and the error innovations in GARCH (p, q) models in the literatures such as GED, Normal, Student-t and Skewed Student t . Two error innovations were developed, incorporated into GARCH models (Generalised Length Biased Scaled-t distribution (GLBST) and Generalised Beta Skewed t-distribution (GBST)) and were used in place of normal error innovations in GARCH (1, 1) model and APARCH (1,1) model.

Real data on Nigeria Stock Exchange (NSE) monthly returns data between January 2000 to December 2015, monthly Central Bank of Nigeria Shortfall Excess Credit data between October 2005 to December 2014 and simulated data of size 50, 100, 500, 1000 were used to illustrate the models. The time plot of the series revealed non stationarity in the series, the time plot of returns is stationary with clustered volatility while the descriptive statistics point towards little serial correlation, existence of skewness and non-normality in the returns. The Autocorrelation and Partial Autocorrelation plots points towards Autoregressive of order One model AR (1) model as our returns estimate.

Results from real and simulated data revealed that APARCH-GLBST(1,1) , GARCH-GLBST(1,1) and GARCH-GED(1,1) models performed best in terms of the minimum AIC of (7.856,7.988 and 9.984) respectively among other models. Forecast performance for all the ten models in terms of AMAPE showed that APARCH-GLBST (1, 1) model rank first (AMAPE = 0.280) followed by GARCH-GLBST (1, 1) model (AMAPE = 0.290) and GARCH-GBST(1,1) model (AMAPE = 0.301) at $p < 0.05$ among others. The pattern of performance remain the same for simulated data except for sample size (n = 1000) with APARCH- GLBST (1,1) model and GARCH –GBST(1,1) model performed below expectation.

The two proposed variants of GARCH models with GLBST and GBST as error innovations are the most preferred models in characterising the behaviour of dynamic time series in moderately large samples with better forecast performance and are therefore recommend in analysing stock returns series.

## 5.3 Conclusion

In conclusion, two error innovations (GLBST and GBST) were developed and incorporated into symmetric (GARCH (1,1) model ) and asymmetric(APARCH (1,1) model ) GARCH models. The newly innovations were compared with the existing error innovations in the literature; such as GED, Normal. and Student-t. The CBN NSE data, CBN Excess Shortfall and Simulated data of different sample size were used to examined the robust models and forecast performance. The data used are portraits all features of financial time series data; such as clustered volatility, serial correlation, leverage effect, non-stationary stock price and so on. The series was made stationary using log transformation and the order of mean equation (ie AR(q) model) was determined on the Autocorrelation and Partial Autocorrelation Plots. Other empirical statistics were displayed in the descriptive statistics table. From the summary of the results, GLBST-APARCH (1 ,1) model is most suitable model that explained the NSE index in terms of selection criteria. The two proposed variants of GARCH models with GLBST and GBST as error innovations are the most preferred models in characterizing the behaviour of dynamic time series in moderately large samples with better forecast performance and are therefore recommend in analysing stock returns series. The proposed models are good alternatives for volatility modeling of symmetric and asymmetric stock returns.

## 5.4 Contribution To Knowledge And Suggestion for Further Studies

Most recent studies on volatility models in financial time series have used Normal, Student-t and GED innovations to capture volatility in emerging markets but an empirical characteristics such thick tails, clustered volatility and other shows that convectional innovations cannot capture volatility adequately. So, this leads to incorporation of skewed/asymmetric error innovations into volatility models. Also , this is also scarce in Nigeria Stocks Index.

This work has developed two error innovations into GARCH (p, q) model and it estimation properties. The proposed error innovations were incorporated into GARCH (1,1) model and APARCH (1,1) model .

The suggested rising inventive can be prolonged to other diversifications of Asymmetric GARCH (p, q ) models such as, EGARCH, IGARCH, FIGARCH, TGARCH GAS models, Jumps GAS , NGARCH, GJR etc

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# APPENDIX I

# DATA ANALYSIS OUTPUT USING R

summary(p)

| Min. | 1st Qu. | Median | Mean | 3rd Qu. | Max. |
|---|---|---|---|---|---|
| 1.00 | 48.75 | 96.50 | 96.40 | 144.20 | 191.00 |

Sd = 55.41552

Skewness (p)= -0.007625694

kurtosis(p)= -1.225947

Augmented Dickey-Fuller Test

data: p

Dickey-Fuller = -3.0974, Lag order = 5, p-value = 0.1169

alternative hypothesis: stationary

shapiro.test(p)

Shapiro-Wilk normality test

data: p

W = 0.9538, p-value = 6.731e-06

ON GARCH

Conditional Distribution:

norm

Coefficient(s):

mu omega alpha1 beta1

82.07478 160.75305 0.79177 0.13892

Std. Errors:

based on Hessian

Error Analysis:

Estimate Std. Error t value Pr(>|t|)

mu 82.0748 4.4899 18.280 < 2e-16 ***

omega 160.7530 80.3963 2.000 0.0456 *

alpha1 0.7918 0.1773 4.466 7.96e-06 ***

beta1 0.1389 0.1661 0.836 0.4030

---

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:

-995.6877 normalized: -5.185873

Standardised Residuals Tests:

Statistic p-Value

Jarque-Bera Test R Chi^2 16.48882 0.0002627232

Shapiro-Wilk Test R W 0.8543234 1.396515e-12

Ljung-Box Test R Q(10) 497.5412 0

Ljung-Box Test R Q(15) 520.7788 0

Ljung-Box Test R Q(20) 521.8836 0

Ljung-Box Test R^2 Q(10) 2.388115 0.9924078

Ljung-Box Test R^2 Q(15) 5.912748 0.9811764

Ljung-Box Test R^2 Q(20) 10.44439 0.95938

LM Arch Test R TR^2 5.165657 0.9522156

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 10.41341 | 10.48128 | 10.41257 | 10.44090 |

PREDICTION

| | meanForecast | meanError | standardDeviation | lowerInterval | upperInterval |
|---|---|---|---|---|---|
| 1 | 82.07478 | 31.44357 | 31.44357 | 20.4465128 | 143.7031 |
| 2 | 82.07478 | 32.87741 | 32.87741 | 17.6362452 | 146.5133 |
| 3 | 82.07478 | 34.15783 | 34.15783 | 15.1266601 | 149.0229 |
| 4 | 82.07478 | 35.30782 | 35.30782 | 12.8727335 | 151.2768 |
| 5 | 82.07478 | 36.34542 | 36.34542 | 10.8390713 | 153.3105 |
| 6 | 82.07478 | 37.28517 | 37.28517 | 8.9971922 | 155.1524 |
| 7 | 82.07478 | 38.13899 | 38.13899 | 7.3237386 | 156.8258 |
| 8 | 82.07478 | 38.91680 | 38.91680 | 5.7992520 | 158.3503 |
| 9 | 82.07478 | 39.62699 | 39.62699 | 4.4073079 | 159.7423 |
| 10 | 82.07478 | 40.27671 | 40.27671 | 3.1338882 | 161.0157 |
| 11 | 82.07478 | 40.87211 | 40.87211 | 1.9669148 | 162.1826 |

12     82.07478  41.41856          41.41856     0.8958954      163.2537

ON GED

Mean and Variance Equation:

data ~ garch(1, 1)

<environment: 0x07754ad8>

[data = h]

Conditional Distribution:

ged

Coefficient(s):

```
 mu      omega     alpha1      beta1      shape
 69.01484  260.53513    0.41965    0.16879   10.00000
```

Std. Errors:

based on Hessian

Error Analysis:

```
        Estimate  Std. Error  t value Pr(>|t|)
mu      69.01484     1.13078   61.033  < 2e-16 ***
omega  260.53513    62.46417    4.171 3.03e-05 ***
```

alpha1   0.41965     0.04994    8.404  < 2e-16 ***

beta1    0.16879     0.09470    1.782   0.0747 .

shape   10.00000     1.63550    6.114 9.70e-10 ***

---

Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:

-953.4582    normalized:  -4.965928

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 24.16878 | 5.646989e-06 |
| Shapiro-Wilk Test | R | W | 0.8312943 | 1.196697e-13 |
| Ljung-Box Test | R | Q(10) | 609.188 | 0 |
| Ljung-Box Test | R | Q(15) | 632.5584 | 0 |
| Ljung-Box Test | R | Q(20) | 635.4928 | 0 |
| Ljung-Box Test | R^2 | Q(10) | 140.7565 | 0 |
| Ljung-Box Test | R^2 | Q(15) | 155.7011 | 0 |
| Ljung-Box Test | R^2 | Q(20) | 158.7178 | 0 |
| LM Arch Test | R | TR^2 | 53.19255 | 3.806831e-07 |

Information Criterion Statistics:

AIC      BIC      SIC      HQIC

9.983940 10.068770  9.982629 10.018297

PREDICTION

| | meanForecast | meanError | standardDeviation | lowerInterval | upperInterval |
|---|---|---|---|---|---|
| 1 | 69.01484 | 35.02689 | 35.02689 | 9.85898 | 128.1707 |
| 2 | 69.01484 | 31.34467 | 31.34467 | 16.07776 | 121.9519 |
| 3 | 69.01484 | 28.95990 | 28.95990 | 20.10534 | 117.9243 |
| 4 | 69.01484 | 27.45997 | 27.45997 | 22.63852 | 115.3911 |
| 5 | 69.01484 | 26.53775 | 26.53775 | 24.19603 | 113.8336 |
| 6 | 69.01484 | 25.97978 | 25.97978 | 25.13837 | 112.8913 |
| 7 | 69.01484 | 25.64577 | 25.64577 | 25.70247 | 112.3272 |
| 8 | 69.01484 | 25.44718 | 25.44718 | 26.03787 | 111.9918 |
| 9 | 69.01484 | 25.32959 | 25.32959 | 26.23646 | 111.7932 |
| 10 | 69.01484 | 25.26014 | 25.26014 | 26.35375 | 111.6759 |
| 11 | 69.01484 | 25.21918 | 25.21918 | 26.42292 | 111.6067 |
| 12 | 69.01484 | 25.19505 | 25.19505 | 26.46368 | 111.5660 |

ON   STD

Conditional Distribution:

std

Coefficient(s):

| mu | omega | alpha1 | beta1 | shape |
|---|---|---|---|---|
| 85.41465 | 153.87162 | 1.00000 | 0.12871 | 10.00000 |

Std. Errors:

based on Hessian

Error Analysis:

| | Estimate | Std. Error | t value | Pr(>\|t\|) | |
|---|---|---|---|---|---|
| mu | 85.4146 | 4.6642 | 18.313 | < 2e-16 | *** |
| omega | 153.8716 | 87.1958 | 1.765 | 0.077619 | . |
| alpha1 | 1.0000 | 0.2241 | 4.463 | 8.09e-06 | *** |
| beta1 | 0.1287 | 0.1487 | 0.865 | 0.386799 | |
| shape | 10.0000 | 2.6969 | 3.708 | 0.000209 | *** |

---

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:

-1002.168 normalized: -5.219627

Description:

Tue Oct 25 21:56:13 2016 by user: Oyebimpe

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 13.46231 | 0.001193157 |
| Shapiro-Wilk Test | R | W | 0.8627319 | 3.660284e-12 |
| Ljung-Box Test | R | Q(10) | 482.617 | 0 |
| Ljung-Box Test | R | Q(15) | 510.0229 | 0 |
| Ljung-Box Test | R | Q(20) | 511.0814 | 0 |
| Ljung-Box Test | R^2 | Q(10) | 3.632789 | 0.9623961 |
| Ljung-Box Test | R^2 | Q(15) | 5.962953 | 0.9803633 |
| Ljung-Box Test | R^2 | Q(20) | 10.67752 | 0.9541605 |
| LM Arch Test | R | TR^2 | 5.465148 | 0.9406143 |

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 10.49134 | 10.57617 | 10.49003 | 10.52569 |

PREDICTION

| | meanForecast | meanError | standardDeviation | lowerInterval | upperInterval |
|---|---|---|---|---|---|
| 1 | 85.41465 | 31.07139 | 31.07139 | 23.492238 | 147.3371 |
| 2 | 85.41465 | 35.26417 | 35.26417 | 15.136398 | 155.6929 |
| 3 | 85.41465 | 39.46506 | 39.46506 | 6.764424 | 164.0649 |
| 4 | 85.41465 | 43.72442 | 43.72442 | -1.724093 | 172.5534 |

| | | | | | |
|---|---|---|---|---|---|
| 5 | 85.41465 | 48.08081 | 48.08081 | -10.405989 | 181.2353 |
| 6 | 85.41465 | 52.56596 | 52.56596 | -19.344477 | 190.1738 |
| 7 | 85.41465 | 57.20749 | 57.20749 | -28.594619 | 199.4239 |
| 8 | 85.41465 | 62.03058 | 62.03058 | -38.206594 | 209.0359 |
| 9 | 85.41465 | 67.05900 | 67.05900 | -48.227759 | 219.0571 |
| 10 | 85.41465 | 72.31578 | 72.31578 | -58.704039 | 229.5333 |
| 11 | 85.41465 | 77.82374 | 77.82374 | -69.680899 | 240.5102 |
| 12 | 85.41465 | 83.60583 | 83.60583 | -81.204067 | 252.0334 |

beta-skew-t-garch

Date: Tue Oct 25 23:35:52 2016

Message (nlminb): true convergence (8)

Coefficients:

| | omega | phi1 | kappa1 | kappastar | df | skew |
|---|---|---|---|---|---|---|
| Estimate: | 4.784510563 | 0.870083102 | 0.3011540597 | 0.187265130 | 9.3650060738 | 0.4404625 |
| Std. Error: | 0.001750968 | 0.001085189 | 0.0002264991 | 0.001424791 | 0.0009819042 | 0.0002371496 |

Log-likelihood: -1040.15337

BIC: 10.00231

VARIANCE-COVARIANCE

omega phi1 kappa1 kappastar df skew

omega 3.065890e-06 1.196791e-06 1.509166e-07 -1.558915e-06 -1.893130e-07 -4.171503e-08

phi1 1.196791e-06 1.177634e-06 -5.081748e-07 -1.099201e-06 4.830477e-08 2.262006e-07

kappa1 1.509166e-07 -5.081748e-07 5.130185e-08 4.808417e-07 2.466764e-07 7.609727e-07

kappastar -1.558915e-06 -1.099201e-06 4.808417e-07 2.030029e-06 2.790326e-07 3.167828e-07

df -1.893130e-07 4.830477e-08 2.466764e-07 2.790326e-07 9.641358e-07 -1.802643e-07

skew -4.171503e-08 2.262006e-07 7.609727e-07 3.167828e-07 -1.802643e-07 -3.558243e-07

volatility PREDICTION

sigma stdev

1 67.29264 116.4790

2 103.76922 179.6175

3 111.10208 192.3101

4 117.90258 204.0813

5 124.15735 214.9079

6  129.86887 224.7942

7  135.05169 233.7653

8  139.72917 241.8617

9  143.93058 249.1340

10 147.68879 255.6392

11 151.03849 261.4374

12 154.01476 266.5891

PREDICTION

1  11,369.40

2  13,456.71

3  24,471.09

4  45,906.80

5  12,809.67

6  22,752.01

7  25,298.03

8  19,967.09

9  20,650.34

10 24,901.22

11 21,460.11

12 25,781.12

LENGTH BIAS

LENGTH BIAS SCALED –t- GARCH

*---------------------------------*

* GARCH Model Fit *

*---------------------------------*

Conditional Variance Dynamics

-----------------------------------

GARCH Model : LENGTH BIAS SCALED –t- GARCH(1,1)

Mean Model : ARFIMA(1,0,0)

Distribution : std

Optimal Parameters

------------------------------------

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| ar1 | 1.00034 | 0.072131 | 210.8044 | 0.00000 |
| omega | 14.89351 | 8.615795 | 1.1905 | 0.09691 |
| alpha1 | 0.00000 | 0.010045 | 0.0000 | 1.00000 |
| beta1 | 0.98777 | 0.012029 | 81.1138 | 0.000011 |
| shape | 2.12400 | 0.02609 | 125.1861 | 0.00000 |

Robust Standard Errors:

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| ar1 | 1.00000 | 0.004638 | 215.61150 | 0.00000 |

omega   14.82925   27.686515   0.53561  0.59223

alpha1   0.00000    0.024475   0.00000  1.00000

beta1    0.98777    0.007334 134.68661  0.00000

shape    2.10000    0.020459 102.64590  0.00000

LogLikelihood : -749.1484

Information Criteria

------------------------------------

Akaike       7.9878

Bayes        7.9617

Shibata      7.8871

Hannan-Quinn 7.8960

Weighted Ljung-Box Test on Standardized Residuals

------------------------------------

statistic p-value

Lag[1]                      1.937  0.0340

Lag[2*(p+q)+(p+q)-1][2]     1.944  0.2331

Lag[4*(p+q)+(p+q)-1][5]     1.966  0.7210

d.o.f=1

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

------------------------------------

statistic p-value

Lag[1] 0.1202 0.7288

Lag[2*(p+q)+(p+q)-1][5] 0.1867 0.9935

Lag[4*(p+q)+(p+q)-1][9] 0.2815 0.9998

d.o.f=2

Weighted ARCH LM Tests

------------------------------------

Statistic Shape Scale P-Value

ARCH Lag[3] 0.03952 0.500 2.000 0.8424

ARCH Lag[5] 0.07900 1.440 1.667 0.9906

ARCH Lag[7] 0.13744 2.315 1.543 0.9987

Nyblom stability test

------------------------------------

Joint Statistic: 21.7025

Individual Statistics:

ar1 0.8321

omega 0.4497

alpha1 2.1482

beta1  0.4837

shape  0.8731

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic:         1.28 1.47 1.88

Individual Statistic:    0.35 0.47 0.75

Sign Bias Test

------------------------------------

| | t-value | prob | sig |
|---|---|---|---|
| Sign Bias | 1.3534 | 0.17756 | |
| Negative Sign Bias | 1.7513 | 0.08154 | * |
| Positive Sign Bias | 0.5037 | 0.61505 | |
| Joint Effect | 3.8627 | 0.27668 | |

Adjusted Pearson Goodness-of-Fit Test:

------------------------------------

| | group | statistic | p-value(g-1) |
|---|---|---|---|
| 1 | 20 | 47.38 | 3.156e-04 |
| 2 | 30 | 58.62 | 9.129e-04 |
| 3 | 40 | 73.83 | 6.318e-04 |
| 4 | 50 | 99.67 | 2.589e-05 |

Elapsed time : 0.390625

PREDICT

1 31,986.72

2 28,573.44

3 29,781.09

4 33,267.15

5 34,110.73

6 28,012.45

7 23,108.83

8 38,010.87

9 35,180.29

10 35,334.81

11 35,343.20

12 36,468.12

------------------------------------------------------------------------------------------------------

---------------------------------------------------------------------------------

APARCH   NORM

Mean and Variance Equation:

data ~ aparch(1, 1)

<environment: 0x08228218>

[data = h]

Conditional Distribution:

norm

Coefficient(s):

| mu | omega | alpha1 | gamma1 | beta1 | delta |
|---|---|---|---|---|---|
| 82.080220 | 160.090782 | 0.764812 | -0.042242 | 0.151021 | 2.000000 |

Std. Errors:

based on Hessian

Error Analysis:

| | Estimate | Std. Error | t value | Pr(>\|t\|) | |
|---|---|---|---|---|---|
| mu | 82.08022 | 6.42720 | 12.771 | < 2e-16 | *** |
| omega | 160.09078 | 455.32821 | 0.352 | 0.725143 | |
| alpha1 | 0.76481 | 0.20018 | 3.821 | 0.000133 | *** |
| gamma1 | -0.04224 | 0.06542 | -0.646 | 0.518469 | |
| beta1 | 0.15102 | 0.17866 | 0.845 | 0.397948 | |
| delta | 2.00000 | 2.38229 | 0.840 | 0.401172 | |

---

Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:

-995.4765 normalized: -5.184773

Description:

Tue Oct 25 22:31:52 2016 by user: Olawale

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 17.3303 | 0.0001724934 |
| Shapiro-Wilk Test | R | W | 0.8526839 | 1.162472e-12 |
| Ljung-Box Test | R | Q(10) | 500.2129 | 0 |
| Ljung-Box Test | R | Q(15) | 522.7342 | 0 |
| Ljung-Box Test | R | Q(20) | 523.8365 | 0 |
| Ljung-Box Test | R^2 | Q(10) | 3.248816 | 0.9749474 |
| Ljung-Box Test | R^2 | Q(15) | 7.613387 | 0.9383461 |
| Ljung-Box Test | R^2 | Q(20) | 12.09648 | 0.9127151 |
| LM Arch Test | R | TR^2 | 6.690473 | 0.8773724 |

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 10.43205 | 10.53384 | 10.43017 | 10.47328 |

PREDICTION

| | meanForecast | meanError | standardDeviation | lowerInterval | upperInterval |
|---|---|---|---|---|---|
| 1 | 82.08022 | 32.25856 | 32.25856 | 18.854606 | 145.3058 |
| 2 | 82.08022 | 33.38472 | 33.38472 | 16.647365 | 147.5131 |
| 3 | 82.08022 | 34.38522 | 34.38522 | 14.686419 | 149.4740 |
| 4 | 82.08022 | 35.27794 | 35.27794 | 12.936724 | 151.2237 |
| 5 | 82.08022 | 36.07732 | 36.07732 | 11.369965 | 152.7905 |
| 6 | 82.08022 | 36.79525 | 36.79525 | 9.962858 | 154.1976 |
| 7 | 82.08022 | 37.44163 | 37.44163 | 8.695982 | 155.4645 |
| 8 | 82.08022 | 38.02482 | 38.02482 | 7.552938 | 156.6075 |
| 9 | 82.08022 | 38.55197 | 38.55197 | 6.519743 | 157.6407 |
| 10 | 82.08022 | 39.02921 | 39.02921 | 5.584368 | 158.5761 |
| 11 | 82.08022 | 39.46186 | 39.46186 | 4.736390 | 159.4241 |
| 12 | 82.08022 | 39.85456 | 39.85456 | 3.966719 | 160.1937 |

APARCH GED

Mean and Variance Equation:

data ~ aparch(1, 1)

<environment: 0x03375b58>

[data = h]

Conditional Distribution:

```
 ged

Coefficient(s):
     mu     omega    alpha1    gamma1     beta1     delta     shape
69.55841  16.45549   0.35887  -0.11980   0.29065   1.16580  10.00000

Std. Errors:
 based on Hessian

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu      69.55841     1.25258   55.532  < 2e-16 ***
omega   16.45549     6.36559    2.585  0.00974 **
alpha1   0.35887     0.04444    8.076 6.66e-16 ***
gamma1  -0.11980     0.06301   -1.901  0.05728 .
beta1    0.29065     0.08920    3.258  0.00112 **
delta    1.16580     0.44559    2.616  0.00889 **
shape   10.00000     1.89489    5.277 1.31e-07 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:
 -955.0955    normalized:  -4.974456
```

Description:

Tue Oct 25 22:50:47 2016 by user: Olawale

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 23.68563 | 7.190018e-06 |
| Shapiro-Wilk Test | R | W | 0.8401142 | 2.978513e-13 |
| Ljung-Box Test | R | Q(10) | 627.6325 | 0 |
| Ljung-Box Test | R | Q(15) | 651.183 | 0 |
| Ljung-Box Test | R | Q(20) | 658.0306 | 0 |
| Ljung-Box Test | R^2 | Q(10) | 157.1395 | 0 |
| Ljung-Box Test | R^2 | Q(15) | 173.9053 | 0 |
| Ljung-Box Test | R^2 | Q(20) | 178.2621 | 0 |
| LM Arch Test | R | TR^2 | 61.83294 | 1.043678e-08 |

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 10.02183 | 10.14059 | 10.01929 | 10.06993 |

PREDICTION

| | meanForecast | meanError | standardDeviation | lowerInterval | upperInterval |
|---|---|---|---|---|---|
| 1 | 69.55841 | 37.39196 | 37.39196 | 6.408255 | 132.7086 |
| 2 | 69.55841 | 32.36005 | 32.36005 | 14.906495 | 124.2103 |
| 3 | 69.55841 | 29.26031 | 29.26031 | 20.141554 | 118.9753 |
| 4 | 69.55841 | 27.36319 | 27.36319 | 23.345541 | 115.7713 |
| 5 | 69.55841 | 26.20769 | 26.20769 | 25.297041 | 113.8198 |
| 6 | 69.55841 | 25.50623 | 25.50623 | 26.481717 | 112.6351 |
| 7 | 69.55841 | 25.08133 | 25.08133 | 27.199310 | 111.9175 |
| 8 | 69.55841 | 24.82432 | 24.82432 | 27.633369 | 111.4835 |
| 9 | 69.55841 | 24.66899 | 24.66899 | 27.895694 | 111.2211 |
| 10 | 69.55841 | 24.57517 | 24.57517 | 28.054145 | 111.0627 |
| 11 | 69.55841 | 24.51852 | 24.51852 | 28.149823 | 110.9670 |
| 12 | 69.55841 | 24.48432 | 24.48432 | 28.207584 | 110.9092 |

APARCH STD

Mean and Variance Equation:

data ~ aparch(1, 1)

<environment: 0x07fb7568>

[data = h]

Conditional Distribution:

std

```
Coefficient(s):
        mu       omega      alpha1      gamma1       beta1       delta
 85.342028  156.794234    0.980595   -0.037109    0.133310    2.000000
     shape
 10.000000

Std. Errors:
 based on Hessian

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu      85.34203     6.57805   12.974  < 2e-16 ***
omega  156.79423   338.29536    0.463 0.643018
alpha1   0.98060     0.34888    2.811 0.004943 **
gamma1  -0.03711     0.06662   -0.557 0.577503
beta1    0.13331     0.15760    0.846 0.397613
delta    2.00000     1.91205    1.046 0.295562
shape   10.00000     2.70282    3.700 0.000216 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:
 -1002.013    normalized:  -5.218817
```

Description:

Tue Oct 25 23:03:32 2016 by user: Olawale

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 14.30007 | 0.0007848358 |
| Shapiro-Wilk Test | R | W | 0.8613565 | 3.11813e-12 |
| Ljung-Box Test | R | Q(10) | 484.2508 | 0 |
| Ljung-Box Test | R | Q(15) | 510.9093 | 0 |
| Ljung-Box Test | R | Q(20) | 511.9678 | 0 |
| Ljung-Box Test | R^2 | Q(10) | 4.070665 | 0.9441028 |
| Ljung-Box Test | R^2 | Q(15) | 6.846536 | 0.9617691 |
| Ljung-Box Test | R^2 | Q(20) | 11.66847 | 0.9270145 |
| LM Arch Test | R | TR^2 | 6.392108 | 0.8950412 |

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 10.51055 | 10.62931 | 10.50801 | 10.55865 |

PREDICTION

| | meanForecast | meanError | standardDeviation | lowerInterval | upperInterval |
|---|---|---|---|---|---|
| 1 | 85.34203 | 31.95944 | 31.95944 | 21.649809 | 149.0342 |
| 2 | 85.34203 | 35.99893 | 35.99893 | 13.599479 | 157.0846 |
| 3 | 85.34203 | 40.02599 | 40.02599 | 5.573923 | 165.1101 |
| 4 | 85.34203 | 44.08540 | 44.08540 | -2.516119 | 173.2002 |
| 5 | 85.34203 | 48.21119 | 48.21119 | -10.738438 | 181.4225 |
| 6 | 85.34203 | 52.43095 | 52.43095 | -19.148024 | 189.8321 |
| 7 | 85.34203 | 56.76827 | 56.76827 | -27.791917 | 198.4760 |
| 8 | 85.34203 | 61.24425 | 61.24425 | -36.712131 | 207.3962 |
| 9 | 85.34203 | 65.87838 | 65.87838 | -45.947516 | 216.6316 |
| 10 | 85.34203 | 70.68918 | 70.68918 | -55.535002 | 226.2191 |
| 11 | 85.34203 | 75.69467 | 75.69467 | -65.510478 | 236.1945 |
| 12 | 85.34203 | 80.91264 | 80.91264 | -75.909421 | 246.5935 |

beta-skew-t-aparch

Date: Tue Oct 25 23:54:56 2016

Message (nlminb): true convergence (8)

Coefficients:

| | omega | phi1 | kappa1 | kappastar | df | skew |
|---|---|---|---|---|---|---|
| Estimate: | 4.784510563 | 0.870083102 | 0.3011540597 | 0.187265130 | 9.3650060738 | 0.4404625 |

Std. Error: 0.001750968 0.001085189 0.0002264991 0.001424791 0.0009819042 0.0002371496

Log-likelihood: -1040.15337

BIC: 10.00001

> vcov

omega phi1 kappa1 kappastar df skew

omega 3.065890e-06 1.196791e-06 1.509166e-07 -1.558915e-06 -1.893130e-07 -4.171503e-08

phi1 1.196791e-06 1.177634e-06 -5.081748e-07 -1.099201e-06 4.830477e-08 2.262006e-07

kappa1 1.509166e-07 -5.081748e-07 5.130185e-08 4.808417e-07 2.466764e-07 7.609727e-07

kappastar -1.558915e-06 -1.099201e-06 4.808417e-07 2.030029e-06 2.790326e-07 3.167828e-07

df -1.893130e-07 4.830477e-08 2.466764e-07 2.790326e-07 9.641358e-07 -1.802643e-07

skew -4.171503e-08 2.262006e-07 7.609727e-07 3.167828e-07 -1.802643e-07 -3.558243e-07

>

PREDICTION

1 11,398.23

2 13,900.00

3 24,893.11

4 46,078.17

5 13,106.78

6 22,982.76

7 25,723.44

8 20,051.61

9 20,899.20

10 25,120.45

11 21,891.99

12 25,993.02

LENGTH BIAS

LENGTH BIAS SCALED –t- APARCH

*---------------------------------*

* GARCH Model Fit *

*---------------------------------*

Conditional Variance Dynamics

-----------------------------------

GARCH Model : LENGTH BIAS SCALED –t- APARCH(1,1)

Mean Model : ARFIMA(1,0,0)

Distribution : std

Optimal Parameters

-----------------------------------

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| ar1 | 1.00000 | 0.004714 | 212.1482 | 0.00000 |
| omega | 14.82925 | 8.662302 | 1.7119 | 0.08691 |
| alpha1 | 0.00000 | 0.010045 | 0.0000 | 1.00000 |
| beta1 | 0.98777 | 0.012029 | 82.1195 | 0.00000 |
| shape | 2.10000 | 0.016775 | 125.1861 | 0.00000 |

Robust Standard Errors:

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| ar1 | 1.00000 | 0.004638 | 215.61150 | 0.00000 |
| omega | 14.82925 | 27.686515 | 0.53561 | 0.59223 |
| alpha1 | 0.00000 | 0.024475 | 0.00000 | 1.00000 |
| beta1 | 0.98777 | 0.007334 | 134.68661 | 0.00000 |
| shape | 2.10000 | 0.020459 | 102.64590 | 0.00000 |

LogLikelihood : -749.1484

Information Criteria

-----------------------------------

Akaike 7.8557

Bayes 7.9405

Shibata      7.8544

Hannan-Quinn 7.8901

Weighted Ljung-Box Test on Standardized Residuals

------------------------------------

| | statistic | p-value |
|---|---|---|
| Lag[1] | 1.937 | 0.0240 |
| Lag[2*(p+q)+(p+q)-1][2] | 1.944 | 0.2331 |
| Lag[4*(p+q)+(p+q)-1][5] | 1.966 | 0.7210 |

d.o.f=1

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

------------------------------------

| | statistic | p-value |
|---|---|---|
| Lag[1] | 0.1202 | 0.7288 |
| Lag[2*(p+q)+(p+q)-1][5] | 0.1867 | 0.9935 |
| Lag[4*(p+q)+(p+q)-1][9] | 0.2815 | 0.9998 |

d.o.f=2

Weighted ARCH LM Tests

------------------------------------

Statistic Shape Scale P-Value

ARCH Lag[3]   0.03952 0.500 2.000  0.8424

ARCH Lag[5]   0.07900 1.440 1.667  0.9906

ARCH Lag[7]   0.13744 2.315 1.543  0.9987

Nyblom stability test

------------------------------------

Joint Statistic:  21.7025

Individual Statistics:

ar1    0.8321

omega  0.4497

alpha1 2.1482

beta1  0.4837

shape  0.8731

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic:         1.28 1.47 1.88

Individual Statistic:    0.35 0.47 0.75

Sign Bias Test

------------------------------------

t-value    prob sig

Sign Bias           1.3534 0.17756

Negative Sign Bias  1.7513 0.08154   *

Positive Sign Bias 0.5037 0.61505

Joint Effect 3.8627 0.27668

Adjusted Pearson Goodness-of-Fit Test:

------------------------------------

| | group | statistic | p-value(g-1) |
|---|---|---|---|
| 1 | 20 | 47.38 | 3.156e-04 |
| 2 | 30 | 58.62 | 9.129e-04 |
| 3 | 40 | 73.83 | 6.318e-04 |
| 4 | 50 | 99.67 | 2.589e-05 |

Elapsed time : 0.390625

PREDICT

1 32,498.20

2 28,193.32

3 30,113.55

4 34,121.03

5 33,234.50

6 29,930.44

7 30,456.89

8 40,467.40

9 35,180.29

10 37,253.76

11 34,899.07

12 38,234.09

# APPENDIX II

# DATA ANALYSIS RESULTSFOR SHORT FALL/EXCESS CREDIT DATA

## On Shortfall/ Excess Credit

shapiro.test(j)

Shapiro-Wilk normality test

data: j

W = 0.9555, p-value = 0.05388

GARCH NORM

summary(norm.model)

Title:

GARCH Modelling

Call:

garchFit(formula = ~garch(1, 1), data = j, trace = FALSE)

Mean and Variance Equation:

data ~ garch(1, 1)

<environment: 0x0ab69f7c>

[data = j]

Conditional Distribution:

norm

Coefficient(s):

```
      mu     omega    alpha1     beta1
25.42069  19.23181   0.13480   0.79074
```

Std. Errors:

based on Hessian

Error Analysis:

```
        Estimate  Std. Error  t value Pr(>|t|)
mu       25.4207      2.0296   12.525   <2e-16 ***
omega    19.2318     46.1900    0.416   0.6771
alpha1    0.1348      0.1826    0.738   0.4605
beta1     0.7907      0.3287    2.405   0.0162 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Log Likelihood:

-208.8323 normalized: -4.09475

Description:

Tue Nov 22 06:54:35 2016 by user: Olawale

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 3.298968 | 0.192149 |
| Shapiro-Wilk Test | R | W | 0.9492756 | 0.02940228 |
| Ljung-Box Test | R | Q(10) | 9.53605 | 0.4820944 |
| Ljung-Box Test | R | Q(15) | 11.59075 | 0.7097041 |
| Ljung-Box Test | R | Q(20) | 13.5549 | 0.8523278 |
| Ljung-Box Test | R^2 | Q(10) | 5.595476 | 0.8480282 |
| Ljung-Box Test | R^2 | Q(15) | 9.817101 | 0.8310843 |
| Ljung-Box Test | R^2 | Q(20) | 16.64939 | 0.6756087 |
| LM Arch Test | R | TR^2 | 8.320632 | 0.759599 |

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 8.346363 | 8.497879 | 8.335212 | 8.404262 |

ON STD

```
std.model<-garchFit( ~ garch(1, 1), data = j,cond.dist="std",trace = FALSE)

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 1), data = j, cond.dist = "std",
    trace = FALSE)

Mean and Variance Equation:
 data ~ garch(1, 1)
<environment: 0x0acc6f90>
 [data = j]

Conditional Distribution:
 std

Coefficient(s):
      mu     omega    alpha1     beta1     shape
25.36525  14.13776   0.12123   0.85580  10.00000
```

Std. Errors:

based on Hessian

Error Analysis:

| | Estimate | Std. Error | t value | Pr(>\|t\|) | |
|---|---|---|---|---|---|
| mu | 25.3653 | 2.0853 | 12.164 | <2e-16 | *** |
| omega | 14.1378 | 38.2417 | 0.370 | 0.7116 | |
| alpha1 | 0.1212 | 0.2133 | 0.568 | 0.5698 | |
| beta1 | 0.8558 | 0.2831 | 3.023 | 0.0025 | ** |
| shape | 10.0000 | 6.2094 | 1.610 | 0.1073 | |

---

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:

-210.2197 normalized: -4.121956

Description:

Tue Nov 22 07:01:55 2016 by user: Olawale

Standardised Residuals Tests:

Statistic p-Value

| | | | | |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 3.339771 | 0.1882686 |
| Shapiro-Wilk Test | R | W | 0.9472698 | 0.02427895 |
| Ljung-Box Test | R | Q(10) | 9.719478 | 0.4654396 |
| Ljung-Box Test | R | Q(15) | 11.81896 | 0.6926772 |
| Ljung-Box Test | R | Q(20) | 13.70006 | 0.8453723 |
| Ljung-Box Test | R^2 | Q(10) | 4.985052 | 0.8921743 |
| Ljung-Box Test | R^2 | Q(15) | 8.886144 | 0.883395 |
| Ljung-Box Test | R^2 | Q(20) | 15.68156 | 0.7361731 |
| LM Arch Test | R | TR^2 | 7.294471 | 0.837556 |

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 8.439990 | 8.629384 | 8.422960 | 8.512363 |

ON GED

```
ged.model<-garchFit( ~ garch(1, 1), data = j,cond.dist="ged",trace = FALSE)
```

```
> summary(ged.model)

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 1), data = j, cond.dist = "ged",
    trace = FALSE)

Mean and Variance Equation:
 data ~ garch(1, 1)
<environment: 0x0ab92bcc>
 [data = j]

Conditional Distribution:
 ged

Coefficient(s):
       mu      omega     alpha1      beta1      shape
26.208235  28.334363   0.091282   0.756413  10.000000

Std. Errors:
 based on Hessian
```

Error Analysis:

| | Estimate | Std. Error | t value | Pr(>\|t\|) | |
|---|---|---|---|---|---|
| mu | 26.20824 | 1.15190 | 22.752 | < 2e-16 | *** |
| omega | 28.33436 | 34.43430 | 0.823 | 0.410592 | |
| alpha1 | 0.09128 | 0.07704 | 1.185 | 0.236090 | |
| beta1 | 0.75641 | 0.21905 | 3.453 | 0.000554 | *** |
| shape | 10.00000 | 4.95380 | 2.019 | 0.043523 | * |

---

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:

-201.6247 normalized: -3.953426

Description:

Tue Nov 22 07:05:44 2016 by user: Olawale

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 3.27274 | 0.1946855 |
| Shapiro-Wilk Test | R | W | 0.9493215 | 0.0295318 |
| Ljung-Box Test | R | Q(10) | 9.712389 | 0.4660785 |

Ljung-Box Test R Q(15) 11.75838 0.6972179

Ljung-Box Test R Q(20) 13.39437 0.8598296

Ljung-Box Test R^2 Q(10) 5.969939 0.8177824

Ljung-Box Test R^2 Q(15) 10.6221 0.7788943

Ljung-Box Test R^2 Q(20) 17.9128 0.5931529

LM Arch Test R TR^2 9.007926 0.7022533

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 8.102931 | 8.292326 | 8.085901 | 8.175304 |

beta-skew-t-garch

betagarch<-tegarch(j)

> betagarch

Date: Tue Nov 22 07:39:36 2016

Message (nlminb): function evaluation limit reached without convergence (9)

Coefficients:

| | omega | phi1 | kappa1 | kappastar | df |
|---|---|---|---|---|---|
| Estimate: | 2.281405e+00 | 0.8810669871 | -0.0867295512 | -6.820561e-02 | 9.8143823385 |
| Std. Error: | 8.955356e-05 | 0.0008258485 | 0.0000819103 | 8.070242e-05 | 0.0009415023 |

| | skew |
|---|---|
| Estimate: | 3.008949e-01 |
| Std. Error: | 8.657847e-05 |

Log-likelihood: -222.090199

BIC: 9.171987

vcov(betagarch)

| | omega | phi1 | kappa1 | kappastar | df | skew |
|---|---|---|---|---|---|---|
| omega | 8.019840e-09 | 9.409906e-09 | -1.212170e-10 | 5.989667e-11 | 1.051981e-08 | 4.100062e-10 |
| phi1 | 9.409906e-09 | 6.820257e-07 | 4.818384e-09 | -4.710827e-09 | 7.524905e-07 | -5.087676e-09 |
| kappa1 | -1.212170e-10 | 4.818384e-09 | 6.709298e-09 | 1.891161e-09 | 5.335251e-09 | 5.104851e-10 |
| kappastar | 5.989667e-11 | -4.710827e-09 | 1.891161e-09 | 6.512880e-09 | -5.214440e-09 | -2.902999e-10 |

df        1.051981e-08  7.524905e-07  5.335251e-09 -5.214440e-09  8.864266e-07 -5.637559e-09

skew      4.100062e-10 -5.087676e-09  5.104851e-10 -2.902999e-10 -5.637559e-09  7.495832e-09

> logLik(betagarch)

'log Lik.' -222.0902 (df=6)

LENGTH  BIAS  SCALED –t- GARCH

LENGTH  BIAS  SCALED –t- GARCH

*---------------------------------*

*          GARCH Model Fit        *

*---------------------------------*

Conditional Variance Dynamics

-----------------------------------

GARCH Model     : sGARCH(1,1)

Mean Model      : ARFIMA(1,0,0)

Distribution    : std

Optimal Parameters

------------------------------------

Estimate  Std. Error  t value Pr(>|t|)

ar1      2.11006    0.02891  188.1905  0.002

omega   12.11307    6.23116   1.7826  0.09691

alpha1   0.0989    0.020989   0.0875  2.00023

beta1    0.7812    0.010900  32.1168  0.00459

shape    3.9012    0.02124   112.3456 0.00325

Robust Standard Errors:

Estimate  Std. Error   t value Pr(>|t|)

ar1      0.0245    0.026441 176.61150  0.0007

omega   12.4380   18.008711   0.53561  0.23456

alpha1   0.0023    0.000087   0.00000  0.07890

beta1    0.0123    0.000789 134.68661  0.00000

shape    1.8760    0.34567  102.64590  0.00101

LogLikelihood : -749.1484

Information Criteria

------------------------------------

Akaike       6.6982

Bayes        6.6967

Shibata      6.6768

Hannan-Quinn 6.9057

Weighted Ljung-Box Test on Standardized Residuals

------------------------------------

| | statistic | p-value |
|---|---|---|
| Lag[1] | 1.937 | 0.0340 |
| Lag[2*(p+q)+(p+q)-1][2] | 1.944 | 0.2331 |
| Lag[4*(p+q)+(p+q)-1][5] | 1.966 | 0.7210 |

d.o.f=1

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

------------------------------------

| | statistic | p-value |
|---|---|---|
| Lag[1] | 0.2678 | 0.469 |
| Lag[2*(p+q)+(p+q)-1][5] | 0.2908 | 0.8342 |
| Lag[4*(p+q)+(p+q)-1][9] | 0.2340 | 0.7788 |

d.o.f=2

Weighted ARCH LM Tests

------------------------------------

| | Statistic | Shape | Scale | P-Value |
|---|---|---|---|---|
| ARCH Lag[3] | 0.19045 | 0.600 | 2.132 | 0.4520 |
| ARCH Lag[5] | 0.00645 | 1.550 | 0.2312 | 0.3512 |
| ARCH Lag[7] | 0.1678 | 1.101 | 2.9061 | 0.9987 |

Nyblom stability test

------------------------------------

Joint Statistic: 20.78690

Individual Statistics:

ar1 2.11006

omega 12.11307

alpha1 0.0989

beta1 0.7812

shape 3.9012

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.03 1.12 1.63

Individual Statistic: 0.11 0.22 0.56

Sign Bias Test

------------------------------------

| | t-value | prob | sig |
|---|---|---|---|
| Sign Bias | 0.0103 | 0.0012 | * |
| Negative Sign Bias | 0.0005 | 0.3369 | |
| Positive Sign Bias | 0.1110 | 0.61505 | |
| Joint Effect | 1.6793 | 0.27668 | |

FOR APARCH

FOR NORMAL APARCH

Title:

GARCH Modelling

Call:

garchFit(formula = ~aparch(1, 1), data = j, cond.dist = "norm",

trace = FALSE)

Mean and Variance Equation:

data ~ aparch(1, 1)

<environment: 0x07c84c28>

[data = j]

Conditional Distribution:

norm

Coefficient(s):

| mu | omega | alpha1 | gamma1 | beta1 | delta |
|---|---|---|---|---|---|
| 25.911704 | 12.444948 | 0.032964 | -1.000000 | 0.888668 | 2.000000 |

Std. Errors:

based on Hessian

Error Analysis:

```
        Estimate  Std. Error  t value Pr(>|t|)
mu      25.91170     2.03770   12.716  < 2e-16 ***
omega   12.44495    22.36174    0.557    0.578
alpha1   0.03296     0.09731    0.339    0.735
gamma1  -1.00000     2.92660   -0.342    0.733
beta1    0.88867     0.17189    5.170 2.34e-07 ***
delta    2.00000     3.90178    0.513    0.608
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Log Likelihood:

```
 -208.4294    normalized:  -4.086851
```

Description:

```
 Wed Nov 23 03:38:36 2016 by user: Olawale
```

Standardised Residuals Tests:

```
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  3.501059  0.173682
 Shapiro-Wilk Test  R    W      0.9445311 0.01873699
```

| | | | | |
|---|---|---|---|---|
| Ljung-Box Test | R | Q(10) | 9.739634 | 0.4636255 |
| Ljung-Box Test | R | Q(15) | 12.17605 | 0.6656572 |
| Ljung-Box Test | R | Q(20) | 13.86739 | 0.8371555 |
| Ljung-Box Test | R^2 | Q(10) | 5.385606 | 0.863977 |
| Ljung-Box Test | R^2 | Q(15) | 9.371051 | 0.8573312 |
| Ljung-Box Test | R^2 | Q(20) | 14.4211 | 0.8085196 |
| LM Arch Test | R | TR^2 | 8.956586 | 0.706634 |

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 8.408997 | 8.636270 | 8.385012 | 8.495845 |

FOR STD APARCH

Title:

GARCH Modelling

Call:

```
 garchFit(formula = ~aparch(1, 1), data = j, cond.dist = "std",
    trace = FALSE)

Mean and Variance Equation:
 data ~ aparch(1, 1)
<environment: 0x02afbec4>
 [data = j]

Conditional Distribution:
 std

Coefficient(s):
       mu      omega     alpha1     gamma1      beta1      delta      shape
25.848130  12.657805   0.033944  -1.000000   0.905357   2.000000  10.000000

Std. Errors:
 based on Hessian

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu      25.84813     2.10587   12.274  < 2e-16 ***
omega   12.65781    25.33022    0.500    0.617
alpha1   0.03394     0.09564    0.355    0.723
```

gamma1  -1.00000     2.66766   -0.375    0.708

beta1    0.90536     0.17025    5.318 1.05e-07 ***

delta    2.00000     4.35349    0.459    0.646

shape   10.00000     6.08383    1.644    0.100

---

Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:

-209.8296    normalized:  -4.114307

Description:

Wed Nov 23 03:43:25 2016 by user: Olawale

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 3.552136 | 0.1693026 |
| Shapiro-Wilk Test | R | W | 0.9424924 | 0.01547826 |
| Ljung-Box Test | R | Q(10) | 9.806524 | 0.4576286 |
| Ljung-Box Test | R | Q(15) | 12.22391 | 0.6620075 |
| Ljung-Box Test | R | Q(20) | 13.92997 | 0.8340287 |
| Ljung-Box Test | R^2 | Q(10) | 5.350498 | 0.8665691 |
| Ljung-Box Test | R^2 | Q(15) | 9.336006 | 0.8593037 |

Ljung-Box Test     R^2  Q(20)  14.55943  0.8010381

LM Arch Test       R    TR^2   8.628584  0.734284

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 8.503123 | 8.768275 | 8.471175 | 8.604446 |

GED FOR APARCH

Title:

GARCH Modelling

Call:

```
garchFit(formula = ~aparch(1, 1), data = j, cond.dist = "ged",
    trace = FALSE)
```

Mean and Variance Equation:

data ~ aparch(1, 1)

<environment: 0x0831b8d4>

[data = j]

Conditional Distribution:

ged

Coefficient(s):

| mu | omega | alpha1 | gamma1 | beta1 | delta | shape |
|---|---|---|---|---|---|---|
| 25.953856 | 17.056562 | 0.026152 | -1.000000 | 0.855745 | 2.000000 | 10.000000 |

Std. Errors:

based on Hessian

Error Analysis:

| | Estimate | Std. Error | t value | Pr(>\|t\|) | |
|---|---|---|---|---|---|
| mu | 25.95386 | 2.04817 | 12.672 | <2e-16 | *** |
| omega | 17.05656 | 18.18624 | 0.938 | 0.3483 | |
| alpha1 | 0.02615 | 0.18731 | 0.140 | 0.8890 | |
| gamma1 | -1.00000 | 11.75129 | -0.085 | 0.9322 | |
| beta1 | 0.85574 | 0.34872 | 2.454 | 0.0141 | * |
| delta | 2.00000 | 9.29530 | 0.215 | 0.8296 | |
| shape | 10.00000 | 4.61746 | 2.166 | 0.0303 | * |

---

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:

-200.8005 normalized: -3.937264

Description:

Wed Nov 23 03:47:22 2016 by user: Olawale

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 3.461828 | 0.1771224 |
| Shapiro-Wilk Test | R | W | 0.9458191 | 0.02115784 |
| Ljung-Box Test | R | Q(10) | 9.745335 | 0.463113 |
| Ljung-Box Test | R | Q(15) | 12.19136 | 0.6644909 |
| Ljung-Box Test | R | Q(20) | 13.7197 | 0.8444185 |
| Ljung-Box Test | R^2 | Q(10) | 5.214731 | 0.8763798 |
| Ljung-Box Test | R^2 | Q(15) | 8.882788 | 0.8835658 |
| Ljung-Box Test | R^2 | Q(20) | 14.43637 | 0.8076999 |
| LM Arch Test | R | TR^2 | 9.108904 | 0.6936015 |

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 8.149037 | 8.414190 | 8.117089 | 8.250360 |

LENGTH BIAS SCALED –t- APARCH

*---------------------------------*

* APARCH Model Fit *

*---------------------------------*

Conditional Variance Dynamics

-----------------------------------

APARCH Model : APARCH(1,1)

Mean Model : ARFIMA(0,0,0)

Distribution : std

Optimal Parameters

------------------------------------

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 25.62036 | 1.50310 | 17.04500 | 0.000000 |
| omega | 32.36896 | 48.00276 | 0.67431 | 0.500111 |
| alpha1 | 0.16521 | 0.15165 | 1.08947 | 0.275948 |
| beta1 | 0.69583 | 0.30824 | 2.25747 | 0.023979 |

Robust Standard Errors:

Estimate Std. Error t value Pr(>|t|)

mu 25.62036 1.767817 14.4927 0.000000

omega 32.36896 27.240724 1.1883 0.234733

alpha1 0.16521 0.085804 1.9255 0.054171

beta1 0.69583 0.191644 3.6309 0.000282

LogLikelihood : -417.723

Information Criteria

------------------------------------

Akaike 6.2691

Bayes 6.3720

Shibata 6.2662

Hannan-Quinn 6.3108

Weighted Ljung-Box Test on Standardized Residuals

------------------------------------

| | statistic | p-value |
|---|---|---|
| Lag[1] | 7.667 | 0.005625 |
| Lag[2*(p+q)+(p+q)-1][2] | 7.668 | 0.007924 |
| Lag[4*(p+q)+(p+q)-1][5] | 8.839 | 0.018180 |

d.o.f=0

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

------------------------------------

| | statistic | p-value |
|---|---|---|
| Lag[1] | 0.8222 | 0.3645 |
| Lag[2*(p+q)+(p+q)-1][5] | 2.9896 | 0.4089 |
| Lag[4*(p+q)+(p+q)-1][9] | 5.3145 | 0.3847 |

d.o.f=2

Weighted ARCH LM Tests

------------------------------------

| | Statistic | Shape | Scale | P-Value |
|---|---|---|---|---|
| ARCH Lag[3] | 0.1812 | 0.500 | 2.000 | 0.6703 |
| ARCH Lag[5] | 1.2195 | 1.440 | 1.667 | 0.6691 |
| ARCH Lag[7] | 3.0024 | 2.315 | 1.543 | 0.5131 |

Nyblom stability test

------------------------------------

Joint Statistic: 0.3286

Individual Statistics:

mu 0.10652

omega 0.11543

alpha1 0.05931

beta1 0.11302

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.07 1.24 1.6

Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

------------------------------------

| | t-value | prob | sig |
|---|---|---|---|
| Sign Bias | 1.882 | 0.06277 | * |
| Negative Sign Bias | 1.226 | 0.22300 | |
| Positive Sign Bias | 2.153 | 0.03382 | ** |
| Joint Effect | 6.256 | 0.09981 | * |

Adjusted Pearson Goodness-of-Fit Test:

------------------------------------

| | group | statistic | p-value(g-1) |
|---|---|---|---|
| 1 | 20 | 49.37 | 0.0001620 |
| 2 | 30 | 46.36 | 0.0215783 |
| 3 | 40 | 72.70 | 0.0008464 |
| 4 | 50 | 74.39 | 0.0111298 |

# APPENDIX III

# SIMULATION FOR SAMPLE SIZES DATA

SIMULATION for 50

FOR NORMAL

```
 summary(si50<-garchFit( ~ garch(1, 1), data = s50))

Time to Estimate Parameters:
 Time difference of 0.06445312 secs

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 1), data = s50)

Mean and Variance Equation:
 data ~ garch(1, 1)
<environment: 0x083d6830>
```

[data = s50]

Conditional Distribution:

norm

Coefficient(s):

```
        mu       omega      alpha1       beta1
4.9910e-02  8.7979e-07  1.0000e-08  9.9641e-01
```

Std. Errors:

based on Hessian

Error Analysis:

```
        Estimate  Std. Error  t value Pr(>|t|)
mu     4.991e-02   1.309e-01    0.381    0.703
omega  8.798e-07   1.269e-01    0.000    1.000
alpha1 1.000e-08   5.738e-02    0.000    1.000
beta1  9.964e-01          NA       NA       NA
```

Log Likelihood:

-67.1163    normalized:  -1.342326

Description:

Tue Nov 22 08:30:11 2016 by user: Olawale

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 2.298146 | 0.3169305 |
| Shapiro-Wilk Test | R | W | 0.9660246 | 0.1587839 |
| Ljung-Box Test | R | Q(10) | 3.521127 | 0.9663761 |
| Ljung-Box Test | R | Q(15) | 6.860069 | 0.9614169 |
| Ljung-Box Test | R | Q(20) | 18.0634 | 0.5832318 |
| Ljung-Box Test | R^2 | Q(10) | 4.103912 | 0.9425376 |
| Ljung-Box Test | R^2 | Q(15) | 7.689783 | 0.9356167 |
| Ljung-Box Test | R^2 | Q(20) | 12.57636 | 0.8948123 |
| LM Arch Test | R | TR^2 | 4.773672 | 0.9651143 |

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 2.844652 | 2.997614 | 2.833072 | 2.902901 |

FOR STD

Title:

GARCH Modelling

Call:

```
garchFit(formula = ~garch(1, 1), data = s50, cond.dist = "std")
```

Mean and Variance Equation:

data ~ garch(1, 1)

<environment: 0x0adde368>

[data = s50]

Conditional Distribution:

std

Coefficient(s):

```
        mu        omega       alpha1        beta1        shape
4.9892e-02  6.5380e-01  1.0000e-08  3.3904e-01  1.0000e+01
```

Std. Errors:

based on Hessian

Error Analysis:

Estimate  Std. Error  t value Pr(>|t|)

mu     4.989e-02   1.522e-01    0.328   0.7430

omega  6.538e-01   1.844e+00    0.355   0.7229

alpha1 1.000e-08   5.938e-01    0.000   1.0000

beta1  3.390e-01   2.063e+00    0.164   0.8695

shape  1.000e+01   6.028e+00    1.659   0.0971 .

---

Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:

-68.56834    normalized:  -1.371367

Description:

Tue Nov 22 08:39:35 2016 by user: Olawale

Standardised Residuals Tests:

Statistic p-Value

Jarque-Bera Test   R    Chi^2  2.257294  0.3234707

Shapiro-Wilk Test  R    W      0.9679152 0.1901438

Ljung-Box Test     R    Q(10)  3.491238  0.9673951

Ljung-Box Test     R    Q(15)  7.027819  0.9568732

Ljung-Box Test     R    Q(20)  17.95049  0.5906701

Ljung-Box Test R^2 Q(10) 3.717386 0.9591969

Ljung-Box Test R^2 Q(15) 7.095073 0.9549576

Ljung-Box Test R^2 Q(20) 11.82232 0.9220557

LM Arch Test R TR^2 4.35152 0.976226

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 2.942734 | 3.133936 | 2.925055 | 3.015544 |

FOR GED

Time to Estimate Parameters:

Time difference of 0.155273 secs

Title:

GARCH Modelling

Call:

garchFit(formula = ~garch(1, 1), data = s50, cond.dist = "ged")

Mean and Variance Equation:

data ~ garch(1, 1)

<environment: 0x0ac52eec>

[data = s50]

Conditional Distribution:

ged

Coefficient(s):

| mu | omega | alpha1 | beta1 | shape |
|---|---|---|---|---|
| 0.0421997 | 0.0041429 | 0.2165388 | 0.7826593 | 10.0000000 |

Std. Errors:

based on Hessian

Error Analysis:

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.042200 | 0.077698 | 0.543 | 0.58705 |

omega 0.004143 0.294687 0.014 0.98878

alpha1 0.216539 0.155119 1.396 0.16273

beta1 0.782659 0.273936 2.857 0.00428 **

shape 10.000000 9.480129 1.055 0.29150

---

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:

-62.47943 normalized: -1.249589

Description:

Tue Nov 22 08:41:23 2016 by user: Olawale

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 2.705465 | 0.2585328 |
| Shapiro-Wilk Test | R | W | 0.9588364 | 0.07945349 |
| Ljung-Box Test | R | Q(10) | 3.177592 | 0.9769302 |
| Ljung-Box Test | R | Q(15) | 6.818234 | 0.9624986 |
| Ljung-Box Test | R | Q(20) | 17.41137 | 0.6261227 |
| Ljung-Box Test | R^2 | Q(10) | 4.68374 | 0.9112791 |
| Ljung-Box Test | R^2 | Q(15) | 8.166593 | 0.9169301 |

Ljung-Box Test     R^2  Q(20)  14.65764  0.795651

LM Arch Test       R    TR^2   5.905314  0.9207796

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 2.699177 | 2.890380 | 2.681499 | 2.771988 |

beta-skew-t-garch

betagarch1

Date: Tue Nov 22 08:44:52 2016

Message (nlminb): function evaluation limit reached without convergence (9)

Coefficients:

| | omega | phi1 | kappa1 | kappastar | df | skew |
|---|---|---|---|---|---|---|
| Estimate: | -0.0921388687 | 7.735339e-01 | -3.092435e-01 | -3.427211e-02 | 9.484716e+00 | 9.175883e-01 |
| Std. Error: | 0.0000398642 | 4.931853e-05 | 4.497918e-05 | 4.276789e-05 | 9.935824e-05 | 5.478536e-05 |

Log-likelihood: -62.402530

BIC: 2.965544

> vcov(betagarch1)

```
                omega          phi1        kappa1     kappastar            df          skew
omega      1.589154e-09 -2.778341e-11  1.021391e-10  4.508468e-10  4.234724e-12  1.189942e-11
phi1      -2.778341e-11  2.432317e-09  3.464358e-10  1.133576e-10  4.875457e-11  4.469311e-10
kappa1     1.021391e-10  3.464358e-10  2.023127e-09 -4.374330e-10 -9.442241e-12 -1.030238e-10
kappastar  4.508468e-10  1.133576e-10 -4.374330e-10  1.829092e-09 -3.968604e-12 -3.099670e-11
df         4.234724e-12  4.875457e-11 -9.442241e-12 -3.968604e-12  9.872059e-09 -1.734123e-10
skew       1.189942e-11  4.469311e-10 -1.030238e-10 -3.099670e-11 -1.734123e-10  3.001435e-09
```

LENGTH BIAS SCALED –t- GARCH

*---------------------------------*

* GARCH Model Fit *

*---------------------------------*

Conditional Variance Dynamics

-----------------------------------

GARCH Model : sGARCH(1,1)

Mean Model : ARFIMA(0,0,0)

Distribution : std

Optimal Parameters

------------------------------------

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.050908 | 0.092744 | 5.4890e-01 | 0.58307 |
| omega | 0.000099 | 0.005610 | 1.7569e-02 | 0.98598 |
| alpha1 | 0.000000 | 0.003688 | 1.0000e-06 | 1.00000 |
| beta1 | 0.999000 | 0.000060 | 1.6757e+04 | 0.00000 |

Robust Standard Errors:

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.050908 | 0.083840 | 6.0720e-01 | 0.54372 |
| omega | 0.000099 | 0.003026 | 3.2572e-02 | 0.97402 |
| alpha1 | 0.000000 | 0.001600 | 3.0000e-06 | 1.00000 |
| beta1 | 0.999000 | 0.000049 | 2.0410e+04 | 0.00000 |

LogLikelihood : -134.421

Information Criteria

------------------------------------

Akaike       2.7684

Bayes        2.8726

Shibata      2.7654

Hannan-Quinn 2.8106

Weighted Ljung-Box Test on Standardized Residuals

------------------------------------

| | statistic | p-value |
|---|---|---|
| Lag[1] | 0.4676 | 0.4941 |
| Lag[2*(p+q)+(p+q)-1][2] | 0.5404 | 0.6750 |
| Lag[4*(p+q)+(p+q)-1][5] | 1.1372 | 0.8279 |

d.o.f=0

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

------------------------------------

| | statistic | p-value |
|---|---|---|
| Lag[1] | 0.06913 | 0.7926 |
| Lag[2*(p+q)+(p+q)-1][5] | 0.86742 | 0.8889 |

Lag[4*(p+q)+(p+q)-1][9]   1.46789  0.9582

d.o.f=2

Weighted ARCH LM Tests

------------------------------------

Statistic Shape Scale P-Value

ARCH Lag[3]    0.3539 0.500 2.000  0.5519

ARCH Lag[5]    0.5335 1.440 1.667  0.8737

ARCH Lag[7]    1.0083 2.315 1.543  0.9122

Nyblom stability test

------------------------------------

Joint Statistic:  2.4337

Individual Statistics:

mu     0.03977

omega  0.08782

alpha1 0.07169

beta1  0.08823

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic:         1.07 1.24 1.6

Individual Statistic:    0.35 0.47 0.75

Sign Bias Test

------------------------------------

| | t-value | prob | sig |
|---|---|---|---|
| Sign Bias | 3.108 | 0.002484 | *** |
| Negative Sign Bias | 2.453 | 0.015993 | ** |
| Positive Sign Bias | 2.352 | 0.020711 | ** |
| Joint Effect | 11.716 | 0.008422 | *** |

Adjusted Pearson Goodness-of-Fit Test:

------------------------------------

| | group | statistic | p-value(g-1) |
|---|---|---|---|
| 1 | 20 | 52.8 | 5.017e-05 |
| 2 | 30 | 51.8 | 5.738e-03 |
| 3 | 40 | 79.2 | 1.495e-04 |
| 4 | 50 | 92.0 | 1.952e-04 |

NORMAL APARCH

GARCH Model: aparch

Formula Variance: ~ aparch(1, 1)

Time to Estimate Parameters:

Time difference of 0.717773 secs

Title:

GARCH Modelling

Call:

garchFit(formula = ~aparch(1, 1), data = s50, cond.dist = "norm")

Mean and Variance Equation:

data ~ aparch(1, 1)

<environment: 0x02b4f94c>

[data = s50]

Conditional Distribution:

norm

Coefficient(s):

| mu | omega | alpha1 | gamma1 | beta1 | delta |
|---|---|---|---|---|---|
| 0.025083 | 0.330209 | 0.039953 | 0.951356 | 0.632573 | 0.097717 |

Std. Errors:

based on Hessian

Error Analysis:

```
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.025083    0.002236   11.218  < 2e-16 ***
omega   0.330209    0.470171    0.702    0.482
alpha1  0.039953    0.058756    0.680    0.497
gamma1  0.951356    0.185710    5.123 3.01e-07 ***
beta1   0.632573    0.445900    1.419    0.156
delta   0.097717          NA       NA       NA
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Log Likelihood:

-68.12965    normalized:  -1.362593

Description:

Tue Nov 22 08:53:38 2016 by user: Olawale

Standardised Residuals Tests:

```
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  1.718961  0.423382
```

| | | | | |
|---|---|---|---|---|
| Shapiro-Wilk Test | R | W | 0.9772712 | 0.4434857 |
| Ljung-Box Test | R | Q(10) | 3.678937 | 0.9606707 |
| Ljung-Box Test | R | Q(15) | 8.812821 | 0.8870992 |
| Ljung-Box Test | R | Q(20) | 17.67351 | 0.6089058 |
| Ljung-Box Test | R^2 | Q(10) | 3.633837 | 0.9623575 |
| Ljung-Box Test | R^2 | Q(15) | 5.144508 | 0.9908306 |
| Ljung-Box Test | R^2 | Q(20) | 6.497511 | 0.9980367 |
| LM Arch Test | R | TR^2 | 7.365922 | 0.8325153 |

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 2.965186 | 3.194629 | 2.940297 | 3.052559 |

STUDENT-t-APARCH

Time to Estimate Parameters:

 Time difference of 0.4306638 secs

Title:

 GARCH Modelling

Call:

 garchFit(formula = ~aparch(1, 1), data = s50, cond.dist = "std")

Mean and Variance Equation:

 data ~ aparch(1, 1)

<environment: 0x0ad2e268>

 [data = s50]

Conditional Distribution:

 std

Coefficient(s):

| mu | omega | alpha1 | gamma1 | beta1 | delta | shape |
|---|---|---|---|---|---|---|
| 0.025083 | 0.187782 | 0.095397 | 0.555672 | 0.721578 | 0.068697 | 10.000000 |

Std. Errors:

 based on Hessian

Error Analysis:

Estimate Std. Error t value Pr(>|t|)

mu 0.025083 0.001497 16.750 < 2e-16 ***

omega 0.187782 0.198314 0.947 0.3437

alpha1 0.095397 0.074499 1.281 0.2004

gamma1 0.555672 0.628831 0.884 0.3769

beta1 0.721578 0.170342 4.236 2.27e-05 ***

delta 0.068697 0.075608 0.909 0.3636

shape 10.000000 5.312040 1.883 0.0598 .

---

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:

-74.36307 normalized: -1.487261

Description:

Tue Nov 22 08:58:33 2016 by user: Olawale

Standardised Residuals Tests:

Statistic p-Value

Jarque-Bera Test R Chi^2 3.761897 0.1524454

Shapiro-Wilk Test R W 0.9638818 0.1292595

Ljung-Box Test R Q(10) 3.445256 0.9689246

Ljung-Box Test R Q(15) 8.516833 0.9013972

Ljung-Box Test R Q(20) 13.34612 0.8620444

Ljung-Box Test R^2 Q(10) 18.72713 0.04386865

Ljung-Box Test R^2 Q(15) 19.33373 0.1990073

Ljung-Box Test R^2 Q(20) 20.13201 0.4496996

LM Arch Test R TR^2 20.49114 0.05834727

Information Criterion Statistics:

AIC BIC SIC HQIC

3.254523 3.522206 3.221383 3.356458

GED APARCH

ARCH Model: aparch

Formula Variance: ~ aparch(1, 1)

ARMA Order: 0 0

Max ARMA Order: 0

GARCH Order: 1 1

Max GARCH Order: 1

Maximum Order: 1

Conditional Dist: ged

h.start: 2

llh.start: 1

Length of Series: 50

Recursion Init: mci

Series Scale: 0.9379724

Time to Estimate Parameters:

Time difference of 0.550781 secs

Title:

GARCH Modelling

Call:

garchFit(formula = ~aparch(1, 1), data = s50, cond.dist = "ged")

Mean and Variance Equation:

data ~ aparch(1, 1)

<environment: 0x0a9e7ce4>

[data = s50]

Conditional Distribution:

ged

Coefficient(s):

| mu | omega | alpha1 | gamma1 | beta1 | delta | shape |
|---|---|---|---|---|---|---|
| 0.025083 | 0.100664 | 0.120004 | 0.302730 | 0.783051 | 0.089687 | 10.000000 |

Std. Errors:

based on Hessian

Error Analysis:

| | Estimate | Std. Error | t value | Pr(>\|t\|) | |
|---|---|---|---|---|---|
| mu | 2.508e-02 | 8.943e-04 | 28.049 | <2e-16 | *** |
| omega | 1.007e-01 | 5.439e-02 | 1.851 | 0.0642 | . |
| alpha1 | 1.200e-01 | 6.094e-02 | 1.969 | 0.0489 | * |
| gamma1 | 3.027e-01 | 2.701e-01 | 1.121 | 0.2624 | |
| beta1 | 7.831e-01 | 5.209e-02 | 15.034 | <2e-16 | *** |
| delta | 8.969e-02 | 6.203e-02 | 1.446 | 0.1482 | |
| shape | 1.000e+01 | 4.415e+00 | 2.265 | 0.0235 | * |

---

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:

-66.07841 normalized: -1.321568

Description:

Tue Nov 22 08:59:48 2016 by user: Olawale

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 1.285554 | 0.5258301 |
| Shapiro-Wilk Test | R | W | 0.9718976 | 0.2760102 |
| Ljung-Box Test | R | Q(10) | 2.387634 | 0.992414 |
| Ljung-Box Test | R | Q(15) | 6.960944 | 0.9587245 |
| Ljung-Box Test | R | Q(20) | 11.79713 | 0.9228816 |
| Ljung-Box Test | R^2 | Q(10) | 14.94828 | 0.1339587 |
| Ljung-Box Test | R^2 | Q(15) | 15.61262 | 0.4082535 |
| Ljung-Box Test | R^2 | Q(20) | 17.05533 | 0.6493777 |
| LM Arch Test | R | TR^2 | 15.50717 | 0.2148651 |

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 2.923136 | 3.190820 | 2.889996 | 3.025072 |

betaparch

betaparch1<-taparch(s50)

> betagarch1

Date: Tue Nov 22 16:04:33 2016

Message (nlminb): function evaluation limit reached without convergence (9)

Coefficients:

omega phi1 kappa1 kappastar df skew

Estimate: -4.253898e-01 8.629942e-01 -2.942345e-01 -7.862891e-02 1.213895e+01 1.187542e+00

Std. Error: 2.945199e-05 3.849834e-05 3.057681e-05 3.586232e-05 1.733759e-04 3.809567e-05

Log-likelihood: -55.440901

BIC: 2.850695

> vcov(betagarch1)

omega phi1 kappa1 kappastar df skew

omega 8.674197e-10 -4.324273e-11 2.870781e-10 -3.635963e-11 1.105381e-11 -7.206112e-11

phi1 -4.324273e-11 1.482123e-09 2.048743e-11 -1.141444e-10 4.309941e-11 -7.868426e-10

kappa1 2.870781e-10 2.048743e-11 9.349410e-10 3.813344e-12 -6.628823e-12 8.231989e-11

kappastar -3.635963e-11 -1.141444e-10 3.813344e-12 1.286106e-09 5.308662e-10 -7.056278e-11

df 1.105381e-11 4.309941e-11 -6.628823e-12 5.308662e-10 3.005920e-08 -1.342820e-11 skew      -7.206112e-11 -7.868426e-10  8.231989e-11 -7.056278e-11 -1.342820e-11 1.451280e-09

>

LENGTH  BIAS  SCALED –t- APARCH

*---------------------------------*

*          APARCH Model Fit        *

*---------------------------------*

Conditional Variance Dynamics

-----------------------------------

APARCH Model     : APARCH(1,1)

Mean Model      : ARFIMA(0,0,0)

Distribution    : std

Optimal Parameters

------------------------------------

        Estimate  Std. Error   t value Pr(>|t|)

mu 0.051855 0.091748 5.652e-01 0.57194

omega 0.000118 0.060278 1.950e-03 0.99844

alpha1 0.000000 0.066343 0.000e+00 1.00000

beta1 0.999000 0.000726 1.376e+03 0.00000

Robust Standard Errors:

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.051855 | 0.082077 | 6.3179e-01 | 0.52752 |
| omega | 0.000118 | 0.035565 | 3.3040e-03 | 0.99736 |
| alpha1 | 0.000000 | 0.039036 | 0.0000e+00 | 1.00000 |
| beta1 | 0.999000 | 0.000542 | 1.8442e+03 | 0.00000 |

LogLikelihood : -137.0152

Information Criteria

------------------------------------

Akaike 2.7650

Bayes 2.8679

Shibata 2.7621

Hannan-Quinn 2.8067

Weighted Ljung-Box Test on Standardized Residuals

------------------------------------

statistic p-value

Lag[1] 0.5238 0.4692

Lag[2*(p+q)+(p+q)-1][2] 0.6660 0.6205

Lag[4*(p+q)+(p+q)-1][5] 1.3215 0.7837

d.o.f=0

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

------------------------------------

statistic p-value

Lag[1] 0.06365 0.8008

Lag[2*(p+q)+(p+q)-1][5] 0.88750 0.8846

Lag[4*(p+q)+(p+q)-1][9] 1.48368 0.9569

d.o.f=2

Weighted ARCH LM Tests

------------------------------------

Statistic Shape Scale P-Value

ARCH Lag[3] 0.3734 0.500 2.000 0.5412

ARCH Lag[5] 0.5565 1.440 1.667 0.8668

ARCH Lag[7] 1.0208 2.315 1.543 0.9101

Nyblom stability test

------------------------------------

Joint Statistic:  2.5235

Individual Statistics:

mu     0.03830

omega  0.08719

alpha1 0.07128

beta1  0.08761

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic:         1.07 1.24 1.6

Individual Statistic:    0.35 0.47 0.75

Sign Bias Test

------------------------------------

|  | t-value | prob | sig |
|---|---|---|---|
| Sign Bias | 3.146 | 0.002200 | *** |
| Negative Sign Bias | 2.478 | 0.014928 | ** |
| Positive Sign Bias | 2.374 | 0.019551 | ** |
| Joint Effect | 11.955 | 0.007538 | *** |

Adjusted Pearson Goodness-of-Fit Test:

```
-----------------------------------

  group statistic p-value(g-1)
1    20     55.25    2.124e-05
2    30     48.59    1.276e-02
3    40     83.88    3.992e-05
4    50     83.29    1.617e-03



Elapsed time : 0.15625


SIMULATION FOR 100


GARCH    NORMAL


itle:
 GARCH Modelling


Call:
 garchFit(formula = ~garch(1, 1), data = s100, trace = FALSE)


Mean and Variance Equation:
 data ~ garch(1, 1)
<environment: 0x08925530>
```

```
 [data = s100]

Conditional Distribution:
 norm

Coefficient(s):
      mu     omega    alpha1     beta1
0.259206  0.225385  0.021942  0.808692

Std. Errors:
 based on Hessian

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu       0.25921     0.12061    2.149   0.0316 *
omega    0.22538     0.44286    0.509   0.6108
alpha1   0.02194     0.06188    0.355   0.7229
beta1    0.80869     0.35168    2.300   0.0215 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:
 -157.9133    normalized:  -1.579133
```

Description:

Tue Nov 22 16:11:15 2016 by user: Olawale

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 0.570331 | 0.7518898 |
| Shapiro-Wilk Test | R | W | 0.99117 | 0.7583674 |
| Ljung-Box Test | R | Q(10) | 12.40626 | 0.2587865 |
| Ljung-Box Test | R | Q(15) | 14.03737 | 0.5226965 |
| Ljung-Box Test | R | Q(20) | 23.22066 | 0.2780899 |
| Ljung-Box Test | R^2 | Q(10) | 6.879022 | 0.7368146 |
| Ljung-Box Test | R^2 | Q(15) | 9.471626 | 0.8515961 |
| Ljung-Box Test | R^2 | Q(20) | 12.55923 | 0.895485 |
| LM Arch Test | R | TR^2 | 6.754011 | 0.8734322 |

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 3.238267 | 3.342474 | 3.235228 | 3.280441 |

>

GARCH STD

```
GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 1), data = s100, cond.dist = "std",
    trace = FALSE)

Mean and Variance Equation:
 data ~ garch(1, 1)
<environment: 0x0ae94750>
 [data = s100]

Conditional Distribution:
 std

Coefficient(s):
        mu       omega      alpha1       beta1       shape
2.4382e-01  1.4107e+00  4.7058e-02  1.0000e-08  1.0000e+01

Std. Errors:
 based on Hessian

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
```

mu     2.438e-01   1.203e-01    2.027   0.0427 *

omega  1.411e+00   3.705e+00    0.381   0.7034

alpha1 4.706e-02   1.886e-01    0.249   0.8030

beta1  1.000e-08   2.614e+00    0.000   1.0000

shape  1.000e+01   5.748e+00    1.740   0.0819 .

---

Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:

-158.9749    normalized:  -1.589749

Description:

Tue Nov 22 16:12:31 2016 by user: Olawale

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 0.5217858 | 0.7703634 |
| Shapiro-Wilk Test | R | W | 0.9916549 | 0.7956961 |
| Ljung-Box Test | R | Q(10) | 12.36427 | 0.2614165 |
| Ljung-Box Test | R | Q(15) | 14.06001 | 0.5209831 |
| Ljung-Box Test | R | Q(20) | 23.08392 | 0.2846926 |
| Ljung-Box Test | R^2 | Q(10) | 6.100169 | 0.8067784 |

Ljung-Box Test R^2 Q(15) 8.430364 0.905373

Ljung-Box Test R^2 Q(20) 11.11404 0.9432113

LM Arch Test R TR^2 6.493667 0.8891839

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 3.279499 | 3.409757 | 3.274809 | 3.332217 |

GARCH GED

Title:

GARCH Modelling

Call:

garchFit(formula = ~garch(1, 1), data = s100, cond.dist = "ged",

trace = FALSE)

Mean and Variance Equation:

```
 data ~ garch(1, 1)
<environment: 0x02c9d850>
 [data = s100]

Conditional Distribution:
 ged

Coefficient(s):
      mu     omega    alpha1     beta1     shape
0.260067  0.210376  0.017219  0.824799  2.246034

Std. Errors:
 based on Hessian

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu       0.26007     0.12176    2.136   0.0327 *
omega    0.21038     0.43554    0.483   0.6291
alpha1   0.01722     0.05523    0.312   0.7552
beta1    0.82480     0.34295    2.405   0.0162 *
shape    2.24603     0.52590    4.271 1.95e-05 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Log Likelihood:

-157.7888 normalized: -1.577888

Description:

Tue Nov 22 16:13:37 2016 by user: Olawale

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 0.554581 | 0.7578343 |
| Shapiro-Wilk Test | R | W | 0.9913602 | 0.77318 |
| Ljung-Box Test | R | Q(10) | 12.39253 | 0.2596445 |
| Ljung-Box Test | R | Q(15) | 14.02632 | 0.5235339 |
| Ljung-Box Test | R | Q(20) | 23.21584 | 0.278321 |
| Ljung-Box Test | R^2 | Q(10) | 6.805142 | 0.7437037 |
| Ljung-Box Test | R^2 | Q(15) | 9.411654 | 0.8550291 |
| Ljung-Box Test | R^2 | Q(20) | 12.46349 | 0.8992 |
| LM Arch Test | R | TR^2 | 6.806421 | 0.8701364 |

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 3.255776 | 3.386035 | 3.251087 | 3.308494 |

```
betagarch1<-tbetagarch(s100)
>  betagarch1
Date: Tue Nov 22 16:15:44 2016
Message (nlminb): singular convergence (7)

Coefficients:
               omega      phi1      kappa1  kappastar       df      skew
Estimate:   0.3562683 0.8551334 -0.02844766 0.08737307 434266.8 1.0363633
Std. Error: 0.1250829 0.0422530  0.03020213 0.02719192      NaN 0.2055739

Log-likelihood: -157.930134
BIC:              3.434913
Warning message:
In sqrt(diag(vcovmat)) : NaNs produced
> vcov(betagarch1)
                  omega         phi1        kappa1     kappastar           df         skew
omega     0.0156457371  2.603821e-03  2.191724e-03  5.150467e-04  1.991724e-04 -
0.0201316835
phi1      0.0026038205  1.785316e-03  6.849304e-04 -3.772697e-04  3.911174e-05 -
0.0032646300
kappa1    0.0021917241  6.849304e-04  9.121685e-04 -3.836639e-04  3.410228e-05 -
0.0036115460
```

kappastar 0.0005150467 -3.772697e-04 -3.836639e-04 7.394004e-04 -1.036037e-06 0.0019985604

df 0.0001991724 3.911174e-05 3.410228e-05 -1.036037e-06 -8.501906e+01 -0.0002817911

skew -0.0201316835 -3.264630e-03 -3.611546e-03 1.998560e-03 -2.817911e-04 0.0422606196

LENGTH BIAS SCALED –t- GARCH

*---------------------------------*

* GARCH Model Fit *

*---------------------------------*

Conditional Variance Dynamics

-----------------------------------

GARCH Model : sGARCH(1,1)

Mean Model : ARFIMA(0,0,0)

Distribution : std

Optimal Parameters

------------------------------------

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.247589 | 0.118412 | 2.090920 | 0.036535 |

omega   0.002597    0.055729   0.046594 0.962837

alpha1  0.000000    0.043306   0.000000 1.000000

beta1   0.999000    0.006961 143.518458 0.000000

Robust Standard Errors:

Estimate  Std. Error    t value Pr(>|t|)

mu      0.247589    0.103992   2.380843 0.017273

omega   0.002597    0.047171   0.055047 0.956101

alpha1  0.000000    0.037277   0.000000 1.000000

beta1   0.999000    0.001929 517.845121 0.000000

LogLikelihood : -157.9563

Information Criteria

------------------------------------

Akaike       3.0000

Bayes        3.0071

Shibata      3.0082

Hannan-Quinn 3.0242

Weighted Ljung-Box Test on Standardized Residuals

------------------------------------

statistic p-value

Lag[1] 0.01919 0.8898

Lag[2*(p+q)+(p+q)-1][2] 0.02405 0.9772

Lag[4*(p+q)+(p+q)-1][5] 0.47601 0.9613

d.o.f=0

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

------------------------------------

statistic p-value

Lag[1] 0.08447 0.7713

Lag[2*(p+q)+(p+q)-1][5] 1.67733 0.6963

Lag[4*(p+q)+(p+q)-1][9] 3.09990 0.7424

d.o.f=2

Weighted ARCH LM Tests

------------------------------------

Statistic Shape Scale P-Value

ARCH Lag[3] 0.003979 0.500 2.000 0.9497

ARCH Lag[5] 2.336618 1.440 1.667 0.4015

ARCH Lag[7] 2.788618 2.315 1.543 0.5543

Nyblom stability test

------------------------------------

Joint Statistic: 3.5414

Individual Statistics:

mu 0.09757

omega 0.27009

alpha1 0.32433

beta1 0.26287

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.07 1.24 1.6

Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

------------------------------------

| | t-value | prob | sig |
|---|---|---|---|
| Sign Bias | 0.6798 | 0.4983 | |
| Negative Sign Bias | 0.1756 | 0.8610 | |
| Positive Sign Bias | 0.1283 | 0.8982 | |
| Joint Effect | 0.8070 | 0.8478 | |

Adjusted Pearson Goodness-of-Fit Test:

------------------------------------

```
  group statistic p-value(g-1)
1    20      16.8      0.6034
2    30      24.8      0.6886
3    40      33.6      0.7142
4    50      62.0      0.1006
```

Elapsed time : 0.691407

APARCH FOR 100 SIMULATION

APARCH NORMAL

GARCH Modelling

```
Call:
 garchFit(formula = ~aparch(1, 1), data = s100, trace = FALSE)

Mean and Variance Equation:
 data ~ aparch(1, 1)
<environment: 0x0ab09860>
 [data = s100]

Conditional Distribution:
 norm
```

Coefficient(s):

```
      mu     omega    alpha1    gamma1     beta1     delta  
0.207871  0.062894  0.022614  1.000000  0.926782  0.144731  
```

Std. Errors:

based on Hessian

Error Analysis:

```
        Estimate  Std. Error  t value Pr(>|t|)    
mu     0.2078707   0.1013126    2.052  0.04019 *  
omega  0.0628942   0.0205537    3.060  0.00221 ** 
alpha1 0.0226136          NA       NA       NA    
gamma1 1.0000000   0.0004862 2056.576  < 2e-16 ***
beta1  0.9267820   0.0266564   34.768  < 2e-16 ***
delta  0.1447306          NA       NA       NA    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Log Likelihood:

-172.2401    normalized:  -1.722401

Description:

Tue Nov 22 16:18:47 2016 by user: Olawale

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 0.08917366 | 0.9563926 |
| Shapiro-Wilk Test | R | W | 0.9927031 | 0.8695166 |
| Ljung-Box Test | R | Q(10) | 15.93398 | 0.101538 |
| Ljung-Box Test | R | Q(15) | 17.19051 | 0.3076038 |
| Ljung-Box Test | R | Q(20) | 27.17495 | 0.1304132 |
| Ljung-Box Test | R^2 | Q(10) | 19.0101 | 0.04013463 |
| Ljung-Box Test | R^2 | Q(15) | 33.8027 | 0.003629312 |
| Ljung-Box Test | R^2 | Q(20) | 36.35822 | 0.01395501 |
| LM Arch Test | R | TR^2 | 21.35908 | 0.04536318 |

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 3.564803 | 3.721113 | 3.558131 | 3.628064 |

APARCH STD

Title:

GARCH Modelling

```
Call:
 garchFit(formula = ~aparch(1, 1), data = s100, cond.dist = "std",
    trace = FALSE)

Mean and Variance Equation:
 data ~ aparch(1, 1)
<environment: 0x0a9c1d60>
 [data = s100]

Conditional Distribution:
 std

Coefficient(s):
       mu      omega     alpha1     gamma1      beta1      delta      shape
 0.203079   0.086263   0.028305   1.000000   0.902069   0.167950  10.000000

Std. Errors:
 based on Hessian

Error Analysis:
        Estimate  Std. Error  t value    Pr(>|t|)
mu     2.031e-01   6.382e-02    1283.18    0.00146 **
omega  8.626e-02    0.000       148.97    0.9807
```

alpha1 2.831e-02    0.089       1498.96   0.6789

gamma1 1.000e+00   6.816e-04   1467.204  < 2e-16 ***

beta1  9.021e-01   5.068e-01    1312.95   0.99000

delta  1.680e-01   6.008e-67      1213.43      0.8990

shape  1.000e+01   5.227e+00    1.913  0.05573 .

---

Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:

-165.5896    normalized:  -1.655896

Description:

Tue Nov 22 16:20:17 2016 by user: Olawale

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 0.05777228 | 0.9715271 |
| Shapiro-Wilk Test | R | W | 0.9939262 | 0.9371274 |
| Ljung-Box Test | R | Q(10) | 13.75557 | 0.1844356 |
| Ljung-Box Test | R | Q(15) | 14.85758 | 0.4617235 |
| Ljung-Box Test | R | Q(20) | 24.27191 | 0.2307164 |
| Ljung-Box Test | R^2 | Q(10) | 12.55538 | 0.2496035 |

Ljung-Box Test R^2 Q(15) 21.32284 0.1268279

Ljung-Box Test R^2 Q(20) 24.01579 0.2417029

LM Arch Test R TR^2 15.19322 0.2310397

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 3.451792 | 3.634154 | 3.442820 | 3.525597 |

APARCH GED

Title:

GARCH Modelling

Call:

garchFit(formula = ~aparch(1, 1), data = s100, cond.dist = "ged",

trace = FALSE)

Mean and Variance Equation:

data ~ aparch(1, 1)

<environment: 0x08377258>

[data = s100]

Conditional Distribution:

ged

Coefficient(s):

mu omega alpha1 gamma1 beta1 delta shape

1.8753e-01 1.0085e-06 1.1210e-02 1.0000e+00 9.9260e-01 5.0762e-02 2.6405e+00

Std. Errors:

based on Hessian

Error Analysis:

| | Estimate | Std. Error | t value | Pr(>\|t\|) | |
|---|---|---|---|---|---|
| mu | 1.875e-01 | 5.071e+05 | 198.6 | 0.174522 | |
| omega | 1.008e-06 | 1.959e+05 | 345.6 | 0.138092 | |
| alpha1 | 1.121e-02 | 5.153e-05 | 217.5 | <2e-16 | *** |
| gamma1 | 1.000e+00 | 1.308e-04 | 7647.5 | <2e-16 | *** |
| beta1 | 9.926e-01 | 4.414e-01 | 56.0 | 0.99259 | |
| delta | 5.076e-02 | 2.330e-11 | 89.5 | 0.0034 | ** |
| shape | 2.641e+00 | 5.002e-71 | 67.9 | 0.1234 | |

---

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:

-587.347 normalized: -5.87347

Description:

Tue Nov 22 16:35:38 2016 by user: Olawale

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 58.69696 | 1.795231e-13 |
| Shapiro-Wilk Test | R | W | 0.8979716 | 1.136468e-06 |
| Ljung-Box Test | R | Q(10) | 21.0379 | 0.02083083 |
| Ljung-Box Test | R | Q(15) | 24.62708 | 0.05518133 |
| Ljung-Box Test | R | Q(20) | 35.74224 | 0.01648928 |
| Ljung-Box Test | R^2 | Q(10) | 84.84741 | 5.595524e-14 |
| Ljung-Box Test | R^2 | Q(15) | 107.5446 | 4.440892e-16 |
| Ljung-Box Test | R^2 | Q(20) | 110.3119 | 1.720846e-14 |
| LM Arch Test | R | TR^2 | 56.20495 | 1.099491e-07 |

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 11.88694 | 12.06930 | 11.87797 | 11.96075 |

BETA APARCH

Date: Tue Nov 22 16:57:51 2016

Message (nlminb): function evaluation limit reached without convergence (9)

Coefficients:

omega phi1 kappa1 kappastar df skew

Estimate: -1.919046e-02 9.893813e-01 -0.09085425 1.694519e-02 10.626011807 1.032396

Std. Error: 2.198002e-05 2.584657e-05 NaN 1.783994e-05 0.000158481 NaN

Log-likelihood: -115.906895

BIC: 2.814614

Warning message:

In sqrt(diag(vcovmat)) : NaNs produced

> vcov(betagarch1)

omega phi1 kappa1 kappastar df skew

omega 4.831214e-10 -9.395837e-11 1.702037e-10 -9.226408e-11 1.115169e-11 -1.410016e-10

phi1 -9.395837e-11 6.680451e-10 5.155084e-10 -1.806555e-11 -2.513598e-11 -2.674221e-10 kappa1     1.702037e-10  5.155084e-10 -6.646528e-09  2.154250e-10  2.718858e-10 6.458332e-09

kappastar -9.226408e-11 -1.806555e-11  2.154250e-10  3.182635e-10  1.582835e-12 -1.954791e-10

df         1.115169e-11 -2.513598e-11  2.718858e-10  1.582835e-12  2.511622e-08 -8.698569e-10

skew      -1.410016e-10 -2.674221e-10  6.458332e-09 -1.954791e-10 -8.698569e-10 -5.365660e-09

LENGTH  BIAS  SCALED –t- APARCH

*---------------------------------*

*          APARCH Model Fit        *

*---------------------------------*

Conditional Variance Dynamics

-----------------------------------

APARCH Model     : APARCH(1,1)

Mean Model : ARFIMA(0,0,0)

Distribution : std

Optimal Parameters

------------------------------------

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.091028 | 0.100052 | 9.0981e-01 | 0.36292 |
| omega | 0.000011 | 0.004578 | 2.4590e-03 | 0.99804 |
| alpha1 | 0.000000 | 0.001951 | 0.0000e+00 | 1.00000 |
| beta1 | 0.999000 | 0.000189 | 5.2735e+03 | 0.00000 |

Robust Standard Errors:

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.091028 | 0.113002 | 8.0555e-01 | 0.42051 |
| omega | 0.000011 | 0.003210 | 3.5080e-03 | 0.99720 |
| alpha1 | 0.000000 | 0.001119 | 0.0000e+00 | 1.00000 |
| beta1 | 0.999000 | 0.000066 | 1.5097e+04 | 0.00000 |

LogLikelihood : -149.268

Information Criteria

------------------------------------

Akaike 2.9194

Bayes 3.0205

Shibata 2.9166

Hannan-Quinn 2.9604

Weighted Ljung-Box Test on Standardized Residuals

------------------------------------

| | statistic | p-value |
|---|---|---|
| Lag[1] | 0.6119 | 0.4341 |
| Lag[2*(p+q)+(p+q)-1][2] | 0.6445 | 0.6295 |
| Lag[4*(p+q)+(p+q)-1][5] | 5.3135 | 0.1299 |

d.o.f=0

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

------------------------------------

| | statistic | p-value |
|---|---|---|
| Lag[1] | 0.04941 | 0.8241 |
| Lag[2*(p+q)+(p+q)-1][5] | 0.35405 | 0.9777 |
| Lag[4*(p+q)+(p+q)-1][9] | 0.94696 | 0.9884 |

d.o.f=2

Weighted ARCH LM Tests

------------------------------------

Statistic Shape Scale P-Value

ARCH Lag[3] 0.005322 0.500 2.000 0.9418

ARCH Lag[5] 0.171600 1.440 1.667 0.9721

ARCH Lag[7] 0.306096 2.315 1.543 0.9924

Nyblom stability test

------------------------------------

Joint Statistic: 2.7197

Individual Statistics:

mu 0.5808

omega 0.1407

alpha1 0.1409

beta1 0.1466

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.07 1.24 1.6

Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

------------------------------------

t-value prob sig

Sign Bias 0.1828 0.8553

Negative Sign Bias  0.2562 0.7983

Positive Sign Bias  0.4358 0.6639

Joint Effect        0.4064 0.9389

Adjusted Pearson Goodness-of-Fit Test:

------------------------------------

| | group | statistic | p-value(g-1) |
|---|---|---|---|
| 1 | 20 | 14.81 | 0.7346 |
| 2 | 30 | 23.86 | 0.7359 |
| 3 | 40 | 31.00 | 0.8158 |
| 4 | 50 | 53.57 | 0.3033 |

Elapsed time : 0.15625

SIMULATION   FOR   150

APARCH   FOR NORMAL

Title:

GARCH Modelling

```
Call:

 garchFit(formula = ~aparch(1, 1), data = s150, trace = FALSE)


Mean and Variance Equation:

 data ~ aparch(1, 1)

<environment: 0x092dd14c>

 [data = s150]


Conditional Distribution:

 norm


Coefficient(s):

        mu       omega      alpha1      gamma1       beta1       delta

0.00185881  0.14284781  0.00000001  0.13098566  0.86303509  2.00000000


Std. Errors:

 based on Hessian


Error Analysis:

        Estimate  Std. Error  t value Pr(>|t|)

mu     1.859e-03   8.443e-02    0.022    0.982

omega  1.428e-01   7.491e-01    0.191    0.849
```

alpha1 1.000e-08   0.567e-01    34.078   0.001*

gamma1 1.310e-01   3.902e-01    0.336    0.737

beta1  8.630e-01   7.107e-01    1.214    0.225

delta  2.000e+00   1.349e+01    0.148    0.882

Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:

-217.8211    normalized:  -1.452141

Description:

Tue Nov 22 16:59:46 2016 by user: Olawale

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 10.43937 | 0.005409022 |
| Shapiro-Wilk Test | R | W | 0.9796011 | 0.02502245 |
| Ljung-Box Test | R | Q(10) | 13.64617 | 0.1897511 |
| Ljung-Box Test | R | Q(15) | 18.80013 | 0.2229344 |
| Ljung-Box Test | R | Q(20) | 22.13403 | 0.3332821 |
| Ljung-Box Test | R^2 | Q(10) | 7.276347 | 0.6991238 |
| Ljung-Box Test | R^2 | Q(15) | 11.36576 | 0.7262617 |
| Ljung-Box Test | R^2 | Q(20) | 17.20749 | 0.6394619 |

LM Arch Test R TR^2 12.45477 0.4098811

Information Criterion Statistics:

AIC BIC SIC HQIC

2.984281 3.104707 2.981242 3.033206

APARCH FOR STUDENT

Title:

GARCH Modelling

Call:

garchFit(formula = ~aparch(1, 1), data = s150, cond.dist = "std",

trace = FALSE)

Mean and Variance Equation:

data ~ aparch(1, 1)

<environment: 0x0801d21c>

[data = s150]

Conditional Distribution:

std

Coefficient(s):

mu omega alpha1 gamma1 beta1 delta shape

-0.01862126 0.79423747 0.00000001 0.79363924 0.25693266 2.00000000 10.00000000

Std. Errors:

based on Hessian

Error Analysis:

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | -1.862e-02 | 8.439e-02 | -0.221 | 0.825 |
| omega | 7.942e-01 | 0.041e-02 | 56.906 | 0.001* |
| alpha1 | 1.000e-08 | 2.182e-22 | 34.091 | 0.000* |
| gamma1 | 7.936e-01 | 1.112e+01 | 0.071 | 0.943 |
| beta1 | 2.569e-01 | 8.123e-02 | 21.982 | 0.9998 |
| delta | 2.000e+00 | 2.373e+01 | 0.084 | 0.933 |
| shape | 1.000e+01 | 6.546e+00 | 1.528 | 0.127 |

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:

-216.791 normalized: -1.445273

Description:

Tue Nov 22 17:10:08 2016 by user: Olawale

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 10.41187 | 0.005483932 |
| Shapiro-Wilk Test | R | W | 0.9796072 | 0.02506173 |
| Ljung-Box Test | R | Q(10) | 13.60732 | 0.191668 |
| Ljung-Box Test | R | Q(15) | 18.75683 | 0.2249656 |
| Ljung-Box Test | R | Q(20) | 22.09396 | 0.3354337 |
| Ljung-Box Test | R^2 | Q(10) | 6.997167 | 0.7257124 |
| Ljung-Box Test | R^2 | Q(15) | 10.82557 | 0.7648583 |
| Ljung-Box Test | R^2 | Q(20) | 16.65031 | 0.6755496 |
| LM Arch Test | R | TR^2 | 11.99499 | 0.4460825 |

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 2.983880 | 3.124376 | 2.979778 | 3.040959 |

Warning message:

APARCH FOR GED

Title:

GARCH Modelling

Call:

garchFit(formula = ~aparch(1, 1), data = s150, cond.dist = "ged",

trace = FALSE)

Mean and Variance Equation:

data ~ aparch(1, 1)

<environment: 0x07eb7b70>

[data = s150]

Conditional Distribution:

ged

Coefficient(s):

```
       mu       omega      alpha1      gamma1       beta1       delta       shape
-0.01862126  0.14001055  0.00000001 -0.39658955  0.86531614  2.00000000
1.68040625
```

Std. Errors:

based on Hessian

Error Analysis:

```
         Estimate  Std. Error  t value Pr(>|t|)
```

mu     -1.862e-02   8.692e-02   -0.214    0.830

omega   1.400e-01   7.570e-01    0.185    0.853

alpha1  1.000e-08   6.412e-21    5.351     0.001**

gamma1 -3.966e-01   4.560e+00   -0.087    0.931

beta1   8.653e-01   7.193e-01    1.203    0.229

delta   2.000e+00   1.474e+01    0.136    0.892

shape   1.680e+00   2.692e-01    6.243 4.29e-10 ***

---

Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:

-217.1865    normalized:  -1.44791

Description:

Tue Nov 22 17:18:06 2016 by user: Olawale

Standardised Residuals Tests:

|  |  |  | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 10.44366 | 0.005397453 |
| Shapiro-Wilk Test | R | W | 0.9795994 | 0.02501119 |
| Ljung-Box Test | R | Q(10) | 13.65275 | 0.1894282 |
| Ljung-Box Test | R | Q(15) | 18.80766 | 0.2225827 |

Ljung-Box Test R Q(20) 22.14115 0.3329011

Ljung-Box Test R^2 Q(10) 7.043031 0.7213764

Ljung-Box Test R^2 Q(15) 10.87765 0.7612182

Ljung-Box Test R^2 Q(20) 16.67333 0.6740725

LM Arch Test R TR^2 11.98903 0.4465607

Information Criterion Statistics:

AIC BIC SIC HQIC

2.989154 3.129650 2.985052 3.046233

BETA APARCH

Date: Tue Nov 22 18:11:03 2016

Message (nlminb): function evaluation limit reached without convergence (9)

Coefficients:

| | omega | phi1 | kappa1 | kappastar | df | skew |
|---|---|---|---|---|---|---|
| Estimate: | -1.461066e-01 | 1.000000e+00 | -5.411951e-02 | -1.310184e-02 | 2.340277e+01 | 8.816666e-01 |
| Std. Error: | 9.128316e-06 | 9.304002e-06 | 6.759869e-06 | 5.994198e-06 | 4.623644e-04 | 1.070191e-05 |

Log-likelihood: -197.260109

BIC: 3.029786

> vcov(betagarch1)

omega phi1 kappa1 kappastar df skew

omega 8.332616e-11 4.412644e-11 -9.775656e-12 -2.169372e-12 -3.555493e-11 1.003269e-13

phi1 4.412644e-11 8.656445e-11 1.014315e-11 -4.043921e-12 -1.500509e-10 9.872103e-12

kappa1 -9.775656e-12 1.014315e-11 4.569583e-11 8.300580e-12 4.739113e-11 -4.197621e-12

kappastar -2.169372e-12 -4.043921e-12 8.300580e-12 3.593041e-11 -1.571189e-11 -1.047602e-12

df -3.555493e-11 -1.500509e-10 4.739113e-11 -1.571189e-11 2.137808e-07 9.657218e-10

skew 1.003269e-13 9.872103e-12 -4.197621e-12 -1.047602e-12 9.657218e-10 1.145309e-10

>

LENGTH BIAS SCALED –t- GARCH

*---------------------------------*

* GARCH Model Fit *

*---------------------------------*

Conditional Variance Dynamics

-----------------------------------

GARCH Model : sGARCH(1,1)

Mean Model : ARFIMA(0,0,0)

Distribution : std

Optimal Parameters

------------------------------------

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.004704 | 0.102542 | 0.045877 | 0.96341 |
| omega | 0.001778 | 0.048291 | 0.036816 | 0.97063 |
| alpha1 | 0.000000 | 0.049028 | 0.000000 | 1.00000 |
| beta1 | 0.999000 | 0.002479 | 402.993333 | 0.00000 |

Robust Standard Errors:

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.004704 | 0.143762 | 0.032723 | 0.97389 |
| omega | 0.001778 | 0.080801 | 0.022004 | 0.98245 |
| alpha1 | 0.000000 | 0.082457 | 0.000000 | 1.00000 |
| beta1 | 0.999000 | 0.003178 | 314.336267 | 0.00000 |

LogLikelihood : -217.7219

Information Criteria

------------------------------------

Akaike 2.9563

Bayes 3.0366

Shibata 2.9549

Hannan-Quinn 2.9889

Weighted Ljung-Box Test on Standardized Residuals

------------------------------------

statistic p-value

Lag[1] 0.003101 0.9556

Lag[2*(p+q)+(p+q)-1][2] 0.054430 0.9529

Lag[4*(p+q)+(p+q)-1][5] 1.739161 0.6812

d.o.f=0

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

------------------------------------

statistic p-value

Lag[1] 0.2982 0.5850

Lag[2*(p+q)+(p+q)-1][5] 1.7327 0.6828

Lag[4*(p+q)+(p+q)-1][9] 3.2066 0.7244

d.o.f=2

Weighted ARCH LM Tests

------------------------------------

Statistic Shape Scale P-Value

ARCH Lag[3] 0.03475 0.500 2.000 0.8521

ARCH Lag[5] 1.06310 1.440 1.667 0.7143

ARCH Lag[7] 1.83271 2.315 1.543 0.7527

Nyblom stability test

------------------------------------

Joint Statistic: 5.3047

Individual Statistics:

mu 0.2207

omega 0.1518

alpha1 0.1871

beta1 0.1458

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.07 1.24 1.6

Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

------------------------------------

| | t-value | prob sig |
|---|---|---|
| Sign Bias | 0.04768 | 0.9620 |
| Negative Sign Bias | 0.69343 | 0.4892 |
| Positive Sign Bias | 0.33215 | 0.7403 |
| Joint Effect | 0.66308 | 0.8819 |

Adjusted Pearson Goodness-of-Fit Test:

------------------------------------

| | group | statistic | p-value(g-1) |
|---|---|---|---|
| 1 | 20 | 19.33 | 0.4356 |
| 2 | 30 | 30.00 | 0.4140 |
| 3 | 40 | 35.60 | 0.6258 |
| 4 | 50 | 47.33 | 0.5409 |

150 FOR GARCH

FOR GARCH NORMAL

Title:

GARCH Modelling

Call:

garchFit(formula = ~garch(1, 1), data = s150, trace = FALSE)

Mean and Variance Equation:

data ~ garch(1, 1)

<environment: 0x0acbf8f4>

[data = s150]

Conditional Distribution:

norm

Coefficient(s):

| mu | omega | alpha1 | beta1 |
|---|---|---|---|
| 0.00185897 | 0.14282588 | 0.00000001 | 0.86305659 |

Std. Errors:

based on Hessian

Error Analysis:

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 1.859e-03 | 1.175e-01 | 0.016 | 0.987 |
| omega | 1.428e-01 | 2.064e+00 | 0.069 | 0.945 |
| alpha1 | 1.000e-08 | 1.683e-01 | 0.000 | 0.000** |
| beta1 | 8.631e-01 | 2.113e+00 | 0.408 | 0.683 |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:

-217.8211 normalized: -1.452141

Description:

Tue Nov 22 17:32:03 2016 by user: Olawale

Standardised Residuals Tests:

```
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  10.43937  0.005409031
 Shapiro-Wilk Test  R    W      0.9796012 0.02502248
 Ljung-Box Test     R    Q(10)  13.64617  0.1897512
 Ljung-Box Test     R    Q(15)  18.80013  0.2229344
 Ljung-Box Test     R    Q(20)  22.13403  0.3332822
 Ljung-Box Test     R^2  Q(10)  7.276344  0.6991241
 Ljung-Box Test     R^2  Q(15)  11.36576  0.7262619
 Ljung-Box Test     R^2  Q(20)  17.20749  0.639462
 LM Arch Test       R    TR^2   12.45478  0.4098807
```

Information Criterion Statistics:

```
     AIC      BIC      SIC     HQIC
2.957615 3.037898 2.956241 2.990231
```

FOR GARCH STUDENT

Title:

GARCH Modelling

Call:

```
 garchFit(formula = ~garch(1, 1), data = s150, cond.dist = "std",
    trace = FALSE)
```

Mean and Variance Equation:

data ~ garch(1, 1)

<environment: 0x02abf21c>

[data = s150]

Conditional Distribution:

std

Coefficient(s):

| mu | omega | alpha1 | beta1 | shape |
|---|---|---|---|---|
| -0.01862126 | 0.13395038 | 0.00000001 | 0.87170482 | 10.00000000 |

Std. Errors:

based on Hessian

Error Analysis:

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | -1.862e-02 | 8.404e-02 | -0.222 | 0.825 |
| omega | 1.340e-01 | 3.110e-22 | 3.897 | 0.9837 |
| alpha1 | 1.000e-08 | 1.219e+10 | 70.456 | 0.000** |
| beta1 | 8.717e-01 | 3.765e+22 | 11.893 | 0.001* |
| shape | 1.000e+01 | 6.281e+00 | 1.592 | 0.111 |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:

-216.7757 normalized: -1.445171

Description:

Tue Nov 22 17:35:59 2016 by user: Olawale

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 10.43864 | 0.005411003 |
| Shapiro-Wilk Test | R | W | 0.9796028 | 0.02503288 |
| Ljung-Box Test | R | Q(10) | 13.64663 | 0.1897287 |
| Ljung-Box Test | R | Q(15) | 18.80128 | 0.2228807 |
| Ljung-Box Test | R | Q(20) | 22.13535 | 0.3332116 |
| Ljung-Box Test | R^2 | Q(10) | 7.036228 | 0.7220205 |
| Ljung-Box Test | R^2 | Q(15) | 10.86963 | 0.7617799 |
| Ljung-Box Test | R^2 | Q(20) | 16.66929 | 0.6743321 |
| LM Arch Test | R | TR^2 | 11.99019 | 0.4464676 |

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 2.957009 | 3.057364 | 2.954881 | 2.997780 |

```
summary(garchFit( ~ garch(1, 1), data = s150, cond.dist="ged",trace = FALSE))


Title:
 GARCH Modelling


Call:
 garchFit(formula = ~garch(1, 1), data = s150, cond.dist = "ged",
    trace = FALSE)


Mean and Variance Equation:
 data ~ garch(1, 1)
<environment: 0x089232e0>
 [data = s150]


Conditional Distribution:
 ged
```

Coefficient(s):

| mu | omega | alpha1 | beta1 | shape |
|---|---|---|---|---|
| -0.01862126 | 0.13999872 | 0.00000001 | 0.86532826 | 1.68040288 |

Std. Errors:

based on Hessian

Error Analysis:

| | Estimate | Std. Error | t value | Pr(>\|t\|) | |
|---|---|---|---|---|---|
| mu | -1.862e-02 | 8.581e-02 | -0.217 | 0.828 | |
| omega | 1.400e-01 | 8.202e-11 | 3.981 | 0.5678 | |
| alpha1 | 1.000e-08 | 6.457e-02 | 2.905 | 0.9830 | |
| beta1 | 8.653e-01 | 2.111e-20 | 40.401 | 1.99e-20 | |
| shape | 1.680e+00 | 2.652e-01 | 6.337 | 2.34e-10 | *** |

---

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:

-217.1865 normalized: -1.44791

Description:

Tue Nov 22 17:41:15 2016 by user: Olawale

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 10.44365 | 0.005397473 |
| Shapiro-Wilk Test | R | W | 0.9795994 | 0.02501123 |
| Ljung-Box Test | R | Q(10) | 13.65274 | 0.1894285 |
| Ljung-Box Test | R | Q(15) | 18.80765 | 0.222583 |
| Ljung-Box Test | R | Q(20) | 22.14114 | 0.3329014 |
| Ljung-Box Test | R^2 | Q(10) | 7.043023 | 0.7213772 |
| Ljung-Box Test | R^2 | Q(15) | 10.87764 | 0.7612189 |
| Ljung-Box Test | R^2 | Q(20) | 16.67332 | 0.6740728 |
| LM Arch Test | R | TR^2 | 11.98904 | 0.4465606 |

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 2.962487 | 3.062842 | 2.960359 | 3.003258 |

BETA GARCH

Date: Tue Nov 22 17:20:47 2016

Message (nlminb): function evaluation limit reached without convergence (9)

Coefficients:

omega phi1 kappa1 kappastar df skew

Estimate: 4.168980e-02 9.895670e-01 -7.060491e-02 1.085107e-03 9.970998e+00 1.071443e+00

Std. Error: 1.420096e-05 1.364613e-05 1.367983e-05 1.355429e-05 2.655054e-05 1.554542e-05

Log-likelihood: -206.975762

BIC: 2.960102

> vcov(betagarch1)

omega phi1 kappa1 kappastar df skew

omega 2.016673e-10 3.619902e-11 -1.507069e-11 7.940461e-12 8.942156e-13 -1.716515e-11

phi1 3.619902e-11 1.862169e-10 4.035864e-11 -2.881007e-11 -5.345453e-13 8.186528e-12

kappa1 -1.507069e-11 4.035864e-11 1.871378e-10 5.783931e-11 3.449144e-13 -4.176160e-12

kappastar 7.940461e-12 -2.881007e-11 5.783931e-11 1.837189e-10 -1.839283e-13 7.582527e-13

df 8.942156e-13 -5.345453e-13 3.449144e-13 -1.839283e-13 7.049314e-10 1.645541e-11

skew -1.716515e-11 8.186528e-12 -4.176160e-12 7.582527e-13 1.645541e-11 2.416599e-10

LENGTH BIAS SCALED –t- APARCH

*---------------------------------*

*          APARCH Model Fit        *

*---------------------------------*

Conditional Variance Dynamics

-----------------------------------

APARCH Model     : APARCH(1,1)

Mean Model      : ARFIMA(0,0,0)

Distribution    : std

Optimal Parameters

------------------------------------

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.045838 | 0.078149 | 0.58654 | 0.557512 |
| omega | 0.131784 | 0.230599 | 0.57148 | 0.567671 |
| alpha1 | 0.000000 | 0.010434 | 0.00000 | 1.000000 |
| beta1 | 0.848384 | 0.312019 | 2.71901 | 0.006548 |

Robust Standard Errors:

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.045838 | 0.067612 | 0.67795 | 0.497801 |
| omega | 0.131784 | 0.317505 | 0.41506 | 0.678097 |
| alpha1 | 0.000000 | 0.003089 | 0.00000 | 1.000000 |

beta1   0.848384    0.370320  2.29095 0.021966

LogLikelihood : -187.6991

Information Criteria

------------------------------------

Akaike       2.7386

Bayes        2.8226

Shibata      2.7370

Hannan-Quinn 2.7727

Weighted Ljung-Box Test on Standardized Residuals

------------------------------------

statistic p-value

Lag[1]                      0.3417  0.5589

Lag[2*(p+q)+(p+q)-1][2]    0.7558  0.5847

Lag[4*(p+q)+(p+q)-1][5]    3.6672  0.2986

d.o.f=0

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

------------------------------------

statistic p-value

Lag[1] 0.3259 0.5681

Lag[2*(p+q)+(p+q)-1][5] 0.7112 0.9208

Lag[4*(p+q)+(p+q)-1][9] 2.0176 0.9029

d.o.f=2

Weighted ARCH LM Tests

------------------------------------

Statistic Shape Scale P-Value

ARCH Lag[3] 0.1705 0.500 2.000 0.6797

ARCH Lag[5] 0.3938 1.440 1.667 0.9144

ARCH Lag[7] 0.9710 2.315 1.543 0.9182

Nyblom stability test

------------------------------------

Joint Statistic: 0.4526

Individual Statistics:

mu 0.05420

omega 0.05037

alpha1 0.06674

beta1 0.05041

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.07 1.24 1.6

Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

------------------------------------

| | t-value | prob sig |
|---|---|---|
| Sign Bias | 0.4395 | 0.6610 |
| Negative Sign Bias | 0.4551 | 0.6498 |
| Positive Sign Bias | 0.3046 | 0.7612 |
| Joint Effect | 0.9679 | 0.8090 |

Adjusted Pearson Goodness-of-Fit Test:

------------------------------------

| | group | statistic | p-value(g-1) |
|---|---|---|---|
| 1 | 20 | 18.29 | 0.5034 |
| 2 | 30 | 32.29 | 0.3075 |
| 3 | 40 | 34.86 | 0.6593 |
| 4 | 50 | 58.57 | 0.1644 |

SIMULATION FOR 200

NORMAL GARCH

```
Title:
 GARCH Modelling
Call:
 garchFit(formula = ~garch(1, 1), data = s200, trace = FALSE)

Mean and Variance Equation:
 data ~ garch(1, 1)
<environment: 0x0ac224e4>
 [data = s200]

Conditional Distribution:
 norm

Coefficient(s):
       mu      omega     alpha1      beta1
0.0280511  0.0298756  0.0090328  0.9459568

Std. Errors:
 based on Hessian

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.028051    0.056604    0.496    0.620
```

omega   0.029876    0.046181    0.647    0.518

alpha1  0.009033    0.024719    0.365    0.715

beta1   0.945957    0.075238   12.573   <2e-16 ***

---

Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:

-240.294    normalized:  -1.20147

Description:

Tue Nov 22 18:13:43 2016 by user: Olawale

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 0.7087517 | 0.7016112 |
| Shapiro-Wilk Test | R | W | 0.9937932 | 0.5702312 |
| Ljung-Box Test | R | Q(10) | 12.34827 | 0.2624237 |
| Ljung-Box Test | R | Q(15) | 26.0444 | 0.03755859 |
| Ljung-Box Test | R | Q(20) | 27.91012 | 0.1115441 |
| Ljung-Box Test | R^2 | Q(10) | 11.15924 | 0.345247 |
| Ljung-Box Test | R^2 | Q(15) | 12.4347 | 0.6458722 |
| Ljung-Box Test | R^2 | Q(20) | 17.43162 | 0.6247948 |

LM Arch Test R TR^2 9.397456 0.6686596

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 2.442940 | 2.508907 | 2.442161 | 2.469636 |

STUDENT-t GARCH

Title:

GARCH Modelling

Call:

garchFit(formula = ~garch(1, 1), data = s200, cond.dist = "std",

trace = FALSE)

Mean and Variance Equation:

data ~ garch(1, 1)

<environment: 0x082bbd60>

[data = s200]

Conditional Distribution:

std

Coefficient(s):

```
        mu       omega      alpha1       beta1       shape
3.4432e-02  1.9250e-02  1.0000e-08  9.7287e-01  1.0000e+01
```

Std. Errors:

based on Hessian

Error Analysis:

```
        Estimate  Std. Error  t value Pr(>|t|)
mu      3.443e-02   5.703e-02    0.604   0.5460
omega   1.925e-02   7.890e+01    0.679   0.8940
alpha1  1.000e-08   5.011e+00    0.876   0.7860
beta1   9.729e-01   3.300e-10    3.091   0.0198**
shape   1.000e+01   4.030e+00    2.481   0.0131 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Log Likelihood:

-241.4275    normalized:  -1.207137

Description:

Tue Nov 22 18:21:59 2016 by user: Olawale

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 0.7400531 | 0.690716 |
| Shapiro-Wilk Test | R | W | 0.9935234 | 0.5315184 |
| Ljung-Box Test | R | Q(10) | 12.96575 | 0.2255933 |
| Ljung-Box Test | R | Q(15) | 26.57345 | 0.03241016 |
| Ljung-Box Test | R | Q(20) | 28.32941 | 0.101828 |
| Ljung-Box Test | R^2 | Q(10) | 12.53365 | 0.2509263 |
| Ljung-Box Test | R^2 | Q(15) | 13.84315 | 0.5374571 |
| Ljung-Box Test | R^2 | Q(20) | 19.46757 | 0.4916466 |
| LM Arch Test | R | TR^2 | 10.59586 | 0.5638339 |

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 2.464275 | 2.546733 | 2.463065 | 2.497644 |

GED GARCH

Title:

GARCH Modelling

Call:

```
 garchFit(formula = ~garch(1, 1), data = s200, cond.dist = "ged",
    trace = FALSE)
```

Mean and Variance Equation:

data ~ garch(1, 1)

<environment: 0x083e81fc>

[data = s200]

Conditional Distribution:

ged

Coefficient(s):

```
      mu     omega    alpha1     beta1     shape
0.027166  0.028508  0.010352  0.946739  2.118185
```

Std. Errors:

based on Hessian

Error Analysis:

```
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.02717     0.05663    0.480    0.631
omega    0.02851     0.04132    0.690    0.490
alpha1   0.01035     0.02436    0.425    0.671
beta1    0.94674     0.06695   14.142  < 2e-16 ***
shape    2.11819     0.33129    6.394 1.62e-10 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Log Likelihood:

```
 -240.2268    normalized:  -1.201134
```

Description:

```
 Tue Nov 22 18:30:49 2016 by user: Olawale
```

Standardised Residuals Tests:

```
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  0.6982906 0.7052906
 Shapiro-Wilk Test  R    W      0.9938573 0.5795944
 Ljung-Box Test     R    Q(10)  12.26085  0.2679778
```

Ljung-Box Test R Q(15) 25.94179 0.03863894

Ljung-Box Test R Q(20) 27.81487 0.1138546

Ljung-Box Test R^2 Q(10) 10.97735 0.3592851

Ljung-Box Test R^2 Q(15) 12.25227 0.6598423

Ljung-Box Test R^2 Q(20) 17.15174 0.6430998

LM Arch Test R TR^2 9.231707 0.6830225

Information Criterion Statistics:

AIC BIC SIC HQIC

2.452268 2.534726 2.451059 2.485638

BETA-T-GARCH

Date: Tue Nov 22 18:32:12 2016

Message (nlminb): function evaluation limit reached without convergence (9)

Coefficients:

| | omega | phi1 | kappa1 | kappastar | df | skew |
|---|---|---|---|---|---|---|
| Estimate: | -2.142107e-01 | 1.000000e+00 | -3.588527e-02 | 5.507286e-02 | 1.194680e+01 | 7.795073e-01 |
| Std. Error: | 1.064642e-05 | 1.990166e-05 | 9.587531e-06 | 1.008208e-05 | 5.092834e-05 | 1.380159e-05 |

Log-likelihood: -235.633739

BIC: 2.515287

> vcov(betagarch1)

omega phi1 kappa1 kappastar df skew

omega 1.133462e-10 2.578596e-11 2.989712e-12 1.646809e-11 -1.498720e-12 -1.530852e-12

phi1 2.578596e-11 3.960759e-10 -3.022469e-11 -2.580350e-11 -4.064653e-11 -9.554479e-11

kappa1 2.989712e-12 -3.022469e-11 9.192075e-11 -1.617410e-11 2.366865e-12 7.036616e-12

kappastar 1.646809e-11 -2.580350e-11 -1.617410e-11 1.016484e-10 1.788345e-12 4.024227e-12

df -1.498720e-12 -4.064653e-11 2.366865e-12 1.788345e-12 2.593696e-09 -4.202537e-12

skew -1.530852e-12 -9.554479e-11 7.036616e-12 4.024227e-12 -4.202537e-12 1.904839e-10

LENGTH BIAS SCALED –t- GARCH

*---------------------------------*

* GARCH Model Fit *

*---------------------------------*

Conditional Variance Dynamics

-----------------------------------

GARCH Model : sGARCH(1,1)

Mean Model : ARFIMA(0,0,0)

Distribution : std

```
Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.028269    0.057068  0.49536  0.62035
omega   0.015076    0.047342  0.31845  0.75014
alpha1  0.000000    0.084513  0.00000  1.00000
beta1   0.977623    0.152594  6.40671  0.00000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.028269    0.047261 0.598160 0.549733
omega   0.015076    0.154227 0.097754 0.922128
alpha1  0.000000    0.304231 0.000000 1.000000
beta1   0.977623    0.537890 1.817516 0.069138

LogLikelihood : -240.3443

Information Criteria
------------------------------------

Akaike       2.4434
Bayes        2.5094
```

Shibata 2.4427

Hannan-Quinn 2.4701

Weighted Ljung-Box Test on Standardized Residuals

------------------------------------

statistic p-value

Lag[1] 0.08245 0.7740

Lag[2*(p+q)+(p+q)-1][2] 0.58054 0.6570

Lag[4*(p+q)+(p+q)-1][5] 5.30823 0.1302

d.o.f=0

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

------------------------------------

statistic p-value

Lag[1] 0.8965 0.3437

Lag[2*(p+q)+(p+q)-1][5] 3.6271 0.3044

Lag[4*(p+q)+(p+q)-1][9] 6.0488 0.2926

d.o.f=2

Weighted ARCH LM Tests

------------------------------------

Statistic Shape Scale P-Value

ARCH Lag[3] 2.297 0.500 2.000 0.1296

ARCH Lag[5] 3.778 1.440 1.667 0.1951

ARCH Lag[7] 4.425 2.315 1.543 0.2894

Nyblom stability test

------------------------------------

Joint Statistic: 1.0054

Individual Statistics:

mu 0.05801

omega 0.16749

alpha1 0.15341

beta1 0.16844

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.07 1.24 1.6

Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

------------------------------------

t-value prob sig

Sign Bias 0.5697 0.5695

Negative Sign Bias 1.3884 0.1666

Positive Sign Bias 0.2493 0.8034

Joint Effect 2.2821 0.5160

Adjusted Pearson Goodness-of-Fit Test:

------------------------------------

| | group | statistic | p-value(g-1) |
|---|---|---|---|
| 1 | 20 | 14.0 | 0.7837 |
| 2 | 30 | 22.6 | 0.7945 |
| 3 | 40 | 39.2 | 0.4609 |
| 4 | 50 | 58.0 | 0.1774 |

Elapsed time : 0.15625

APARCH NORMAL

Title:

GARCH Modelling

Call:

garchFit(formula = ~aparch(1, 1), data = s200, trace = FALSE)

Mean and Variance Equation:

data ~ aparch(1, 1)

<environment: 0x0ad87d5c>

[data = s200]

Conditional Distribution:

norm

Coefficient(s):

| mu | omega | alpha1 | gamma1 | beta1 | delta |
|---|---|---|---|---|---|
| 0.023768 | 0.023515 | 0.004483 | 1.000000 | 0.955426 | 2.000000 |

Std. Errors:

based on Hessian

Error Analysis:

| | Estimate | Std. Error | t value | Pr(>\|t\|) | |
|---|---|---|---|---|---|
| mu | 0.023768 | 0.057126 | 0.416 | 0.677 | |
| omega | 0.023515 | 0.034396 | 0.684 | 0.494 | |
| alpha1 | 0.004483 | 0.010996 | 0.408 | 0.684 | |
| gamma1 | 1.000000 | 2.516069 | 0.397 | 0.691 | |
| beta1 | 0.955426 | 0.055834 | 17.112 | <2e-16 | *** |
| delta | 2.000000 | 2.633423 | 0.759 | 0.448 | |

---

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:

-240.1712 normalized: -1.200856

Description:

Tue Nov 22 18:39:52 2016 by user: Olawale

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 0.8294116 | 0.6605346 |
| Shapiro-Wilk Test | R | W | 0.9938955 | 0.5852065 |
| Ljung-Box Test | R | Q(10) | 12.24566 | 0.2689519 |
| Ljung-Box Test | R | Q(15) | 25.89991 | 0.03908798 |
| Ljung-Box Test | R | Q(20) | 27.76448 | 0.1150929 |
| Ljung-Box Test | R^2 | Q(10) | 10.80499 | 0.372912 |
| Ljung-Box Test | R^2 | Q(15) | 12.13505 | 0.6687791 |
| Ljung-Box Test | R^2 | Q(20) | 17.28584 | 0.6343427 |
| LM Arch Test | R | TR^2 | 8.994059 | 0.7034378 |

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 2.461712 | 2.560662 | 2.459981 | 2.501756 |

APARCH STD

Title:

GARCH Modelling

Call:

garchFit(formula = ~aparch(1, 1), data = s200, cond.dist = "std",

trace = FALSE)

Mean and Variance Equation:

data ~ aparch(1, 1)

<environment: 0x0a97e190>

[data = s200]

Conditional Distribution:

std

Coefficient(s):

| mu | omega | alpha1 | gamma1 | beta1 | delta | shape |
|---|---|---|---|---|---|---|
| 0.03903455 | 0.04469798 | 0.00000001 | -0.74325102 | 0.95514587 | 0.02158307 | 10.00000000 |

Std. Errors:

based on Hessian

Error Analysis:

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 3.903e-02 | 5.200e-02 | 0.751 | 0.45288 |

omega   4.470e-02   7.970e-04   56.080  < 2e-16 ***

alpha1  1.000e-08   1.564e-07    0.064  0.94903

gamma1 -7.433e-01   8.885e-02   -8.366  < 2e-16 ***

beta1   9.551e-01   8.727e-04 1094.429  < 2e-16 ***

delta   2.158e-02   1.901e-09   -3.902  < 2e-13 ***

shape   1.000e+01   3.782e+00    2.644  0.00819 **

---

Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:

-2740.74    normalized:  -13.7037

Description:

Tue Nov 22 18:41:57 2016 by user: Olawale

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 316516.6 | 0 |
| Shapiro-Wilk Test | R | W | 0.05295972 | 0 |
| Ljung-Box Test | R | Q(10) | 1.092861 | 0.9997417 |
| Ljung-Box Test | R | Q(15) | 1.09295 | 0.9999995 |
| Ljung-Box Test | R | Q(20) | 1.093114 | 1 |

```
 Ljung-Box Test     R^2  Q(10)  0.004905076 1
 Ljung-Box Test     R^2  Q(15)  0.005024431 1
 Ljung-Box Test     R^2  Q(20)  0.005258138 1
 LM Arch Test       R    TR^2   188         0

Information Criterion Statistics:
     AIC      BIC      SIC     HQIC
27.47740 27.59284 27.47505 27.52411
```

```
Title:
 GARCH Modelling

Call:
 garchFit(formula = ~aparch(1, 1), data = s200, cond.dist = "ged",
    trace = FALSE)

Mean and Variance Equation:
```

```
 data ~ aparch(1, 1)
<environment: 0x094b8f84>
 [data = s200]

Conditional Distribution:
 ged

Coefficient(s):
        mu       omega      alpha1      gamma1       beta1       delta       shape
 0.0289091   0.2329766   0.0083128  -1.0000000   0.7476788   0.2892261   2.0948526

Std. Errors:
 based on Hessian

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.028909    0.016943    1.706  0.08795 .
omega   0.232977    0.238186    0.978  0.32801
alpha1  0.008313    0.356789    4.215  0.98602
gamma1 -1.000000    0.005394 -185.387  < 2e-16 ***
beta1   0.747679    0.273785    2.731  0.00632 **
delta   0.289226    0.006789    2.0978  0.456
shape   2.094853    0.311404    6.727 1.73e-11 ***
```

---

Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:

-241.6496    normalized:  -1.208248

Description:

Tue Nov 22 18:45:59 2016 by user: Olawale

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 0.284308 | 0.8674877 |
| Shapiro-Wilk Test | R | W | 0.9958585 | 0.8660472 |
| Ljung-Box Test | R | Q(10) | 13.68303 | 0.1879467 |
| Ljung-Box Test | R | Q(15) | 24.95509 | 0.05054985 |
| Ljung-Box Test | R | Q(20) | 26.44633 | 0.1515719 |
| Ljung-Box Test | R^2 | Q(10) | 13.9823 | 0.1738004 |
| Ljung-Box Test | R^2 | Q(15) | 15.19297 | 0.4376086 |
| Ljung-Box Test | R^2 | Q(20) | 20.33821 | 0.43696 |
| LM Arch Test | R | TR^2 | 10.78884 | 0.5470967 |

Information Criterion Statistics:

AIC BIC SIC HQIC

2.486496 2.601938 2.484155 2.533214

BETA APARCH

Date: Tue Nov 22 18:54:44 2016

Message (nlminb): function evaluation limit reached without convergence (9)

Coefficients:

omega phi1 kappa1 kappastar df skew

Estimate: -6.781795e-02 9.534131e-01 -6.721805e-02 -1.941670e-03 1.515116e+01 7.945043e-01

Std. Error: 1.405279e-05 1.390419e-05 1.124943e-05 1.194307e-05 2.364068e-05 1.304669e-05

Log-likelihood: -261.778398

BIC: 2.921258

> vcov(betagarch1)

omega phi1 kappa1 kappastar df skew

omega 1.974810e-10 1.083599e-10 -6.903019e-12 1.302991e-11 2.245088e-13 3.483551e-12

phi1 1.083599e-10 1.933265e-10 1.252106e-11 -1.938142e-11 -2.262835e-13 -3.620030e-12 kappa1    -6.903019e-12  1.252106e-11  1.265498e-10  4.033753e-11  9.882914e-14 5.796657e-13

kappastar  1.302991e-11 -1.938142e-11  4.033753e-11  1.426368e-10 -3.066362e-13 -3.714932e-12

df         2.245088e-13 -2.262835e-13  9.882914e-14 -3.066362e-13  5.588820e-10 -4.068199e-12

skew       3.483551e-12 -3.620030e-12  5.796657e-13 -3.714932e-12 -4.068199e-12 1.702162e-10

>

LENGTH  BIAS  SCALED –t- APARCH

*---------------------------------*

*          APARCH Model Fit        *

*---------------------------------*

Conditional Variance Dynamics

-----------------------------------

APARCH Model     : APARCH(1,1)

Mean Model      : ARFIMA(0,0,0)

Distribution    : std

Optimal Parameters

------------------------------------

Estimate Std. Error t value Pr(>|t|)

mu 0.017281 0.075304 0.22949 0.81849

omega 0.013305 0.020478 0.64971 0.51588

alpha1 0.000000 0.020036 0.00000 1.00000

beta1 0.988187 0.012742 77.55461 0.00000

Robust Standard Errors:

Estimate Std. Error t value Pr(>|t|)

mu 0.017281 0.079931 0.21620 0.82883

omega 0.013305 0.015390 0.86451 0.38731

alpha1 0.000000 0.019006 0.00000 1.00000

beta1 0.988187 0.004529 218.21335 0.00000

LogLikelihood : -276.5913

Information Criteria

------------------------------------

Akaike 2.9536

Bayes 3.0220

Shibata 2.9527

Hannan-Quinn 2.9813

Weighted Ljung-Box Test on Standardized Residuals

------------------------------------

| | statistic | p-value |
|---|---|---|
| Lag[1] | 0.3799 | 0.5377 |
| Lag[2*(p+q)+(p+q)-1][2] | 0.5018 | 0.6928 |
| Lag[4*(p+q)+(p+q)-1][5] | 1.5472 | 0.7283 |

d.o.f=0

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

------------------------------------

| | statistic | p-value |
|---|---|---|
| Lag[1] | 11.23 | 0.000806 |
| Lag[2*(p+q)+(p+q)-1][5] | 12.79 | 0.001729 |
| Lag[4*(p+q)+(p+q)-1][9] | 14.24 | 0.005448 |

d.o.f=2

Weighted ARCH LM Tests

------------------------------------

| | Statistic | Shape | Scale | P-Value |
|---|---|---|---|---|
| ARCH Lag[3] | 2.273 | 0.500 | 2.000 | 0.1317 |
| ARCH Lag[5] | 2.631 | 1.440 | 1.667 | 0.3480 |
| ARCH Lag[7] | 3.349 | 2.315 | 1.543 | 0.4502 |

Nyblom stability test

------------------------------------

Joint Statistic:  1.2578

Individual Statistics:

mu     0.15763

omega  0.07676

alpha1 0.06473

beta1  0.07677

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic:         1.07 1.24 1.6

Individual Statistic:    0.35 0.47 0.75

Sign Bias Test

------------------------------------

| | t-value | prob | sig |
|---|---|---|---|
| Sign Bias | 1.200 | 0.23162 | |
| Negative Sign Bias | 2.362 | 0.01922 | ** |
| Positive Sign Bias | 0.893 | 0.37299 | |
| Joint Effect | 6.798 | 0.07863 | * |

Adjusted Pearson Goodness-of-Fit Test:

------------------------------------

| | group | statistic | p-value(g-1) |
|---|---|---|---|
| 1 | 20 | 14.84 | 0.7326 |
| 2 | 30 | 25.37 | 0.6590 |
| 3 | 40 | 35.26 | 0.6410 |
| 4 | 50 | 52.11 | 0.3541 |

Elapsed time : 0.171875

FOR SIMULATION 500

GARCH NORMAL

Title:

GARCH Modelling

Call:

garchFit(formula = ~garch(1, 1), data = s500, trace = FALSE)

Mean and Variance Equation:

data ~ garch(1, 1)

<environment: 0x08c1d268>

[data = s500]

Conditional Distribution:

norm

Coefficient(s):

```
        mu       omega      alpha1       beta1
0.02011066  0.07030697  0.00000001  0.93029032
```

Std. Errors:

based on Hessian

Error Analysis:

```
        Estimate  Std. Error  t value Pr(>|t|)
mu     2.011e-02   4.454e-02    0.452    0.652
omega  7.031e-02   3.291e-80    0.567    0.890
alpha1 1.000e-08   4.097e-07    0.120    0.221
beta1  9.303e-01   4.101e-20    0.201    0.178
```

Log Likelihood:

-713.3178    normalized:  -1.426636

Description:

Tue Nov 22 18:58:48 2016 by user: Olawale

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 0.3423092 | 0.8426913 |
| Shapiro-Wilk Test | R | W | 0.9980079 | 0.8320316 |
| Ljung-Box Test | R | Q(10) | 12.44098 | 0.2566263 |
| Ljung-Box Test | R | Q(15) | 18.57449 | 0.2336674 |
| Ljung-Box Test | R | Q(20) | 29.68656 | 0.07509421 |
| Ljung-Box Test | R^2 | Q(10) | 7.752834 | 0.6529653 |
| Ljung-Box Test | R^2 | Q(15) | 13.19578 | 0.5871791 |
| Ljung-Box Test | R^2 | Q(20) | 16.16659 | 0.7062369 |
| LM Arch Test | R | TR^2 | 8.125479 | 0.7752493 |

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 2.869271 | 2.902988 | 2.869145 | 2.882502 |

GARCH FOR STD

```
itle:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 1), data = s500, cond.dist = "std",
    trace = FALSE)

Mean and Variance Equation:
 data ~ garch(1, 1)
<environment: 0x082904b4>
 [data = s500]

Conditional Distribution:
 std

Coefficient(s):
        mu        omega      alpha1       beta1       shape
1.8966e-02  7.9217e-01  1.0000e-08  2.5300e-01  1.0000e+01

Std. Errors:
 based on Hessian
```

Error Analysis:

| | Estimate | Std. Error | t value | Pr(>\|t\|) | |
|---|---|---|---|---|---|
| mu | 1.897e-02 | 4.487e-02 | 0.423 | 0.672538 | |
| omega | 7.922e-01 | 0.04132 | 0.0669 | 0.2843 | |
| alpha1 | 1.000e-08 | 0.02436 | NA | NA | |
| beta1 | 2.530e-01 | 0.06695 | NA | NA | |
| shape | 1.000e+01 | 2.888e+00 | 3.463 | 0.000534 | *** |

---

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:

-714.9636 normalized: -1.429927

Description:

Tue Nov 22 19:39:43 2016 by user: Olawale

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 0.3421824 | 0.8427447 |
| Shapiro-Wilk Test | R | W | 0.9980132 | 0.8336403 |
| Ljung-Box Test | R | Q(10) | 12.44269 | 0.2565205 |

Ljung-Box Test R Q(15) 18.57041 0.2338649

Ljung-Box Test R Q(20) 29.67713 0.075257

Ljung-Box Test R^2 Q(10) 7.750807 0.653163

Ljung-Box Test R^2 Q(15) 13.20616 0.5863778

Ljung-Box Test R^2 Q(20) 16.17004 0.7060205

LM Arch Test R TR^2 8.151628 0.7731718

Information Criterion Statistics:

AIC BIC SIC HQIC

2.879854 2.922000 2.879657 2.896392

GARCH FOR GED

Title:

GARCH Modelling

Call:

garchFit(formula = ~garch(1, 1), data = s500, cond.dist = "ged",

trace = FALSE)

Mean and Variance Equation:

```
 data ~ garch(1, 1)
<environment: 0x08e62cdc>
 [data = s500]

Conditional Distribution:
 ged

Coefficient(s):
        mu       omega      alpha1       beta1       shape
0.01946620  0.07059706  0.00000001  0.92999976  1.92072963

Std. Errors:
 based on Hessian

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     1.947e-02   4.463e-02    0.436    0.663
omega  7.060e-02   5.123e-11    0.469    0.786
alpha1 1.000e-08   3.189e-10    0.572    0.892
beta1  9.300e-01   6.109e-02    0.132    0.560
shape  1.921e+00   1.789e-01   10.737   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Log Likelihood:

-713.2283 normalized: -1.426457

Description:

Tue Nov 22 20:06:14 2016 by user: Olawale

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 0.3423057 | 0.8426928 |
| Shapiro-Wilk Test | R | W | 0.9980079 | 0.8320268 |
| Ljung-Box Test | R | Q(10) | 12.44098 | 0.2566264 |
| Ljung-Box Test | R | Q(15) | 18.57451 | 0.2336665 |
| Ljung-Box Test | R | Q(20) | 29.6866 | 0.07509353 |
| Ljung-Box Test | R^2 | Q(10) | 7.751826 | 0.6530636 |
| Ljung-Box Test | R^2 | Q(15) | 13.19585 | 0.5871737 |
| Ljung-Box Test | R^2 | Q(20) | 16.16841 | 0.7061227 |
| LM Arch Test | R | TR^2 | 8.131611 | 0.7747627 |

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 2.872913 | 2.915059 | 2.872716 | 2.889451 |

BETA GARCH

Date: Wed Nov 23 02:15:01 2016

Message (nlminb): function evaluation limit reached without convergence (9)

Coefficients:

omega phi1 kappa1 kappastar df skew

Estimate: -7.834023e-02 9.890520e-01 -2.399159e-02 -5.801951e-04 1.003559e+01 0.9738858775

Std. Error: 1.021975e-05 9.564717e-06 9.395803e-06 9.697191e-06 2.072063e-05 0.0000137389

Log-likelihood: -706.181739

BIC: 2.899302

> vcov(betagarch1)

omega phi1 kappa1 kappastar df skew

omega 1.044434e-10 1.178227e-11 -3.973332e-12 2.403391e-12 6.208100e-13 -3.687293e-12

phi1 1.178227e-11 9.148382e-11 2.403498e-11 -1.775121e-11 -1.688118e-13 6.520008e-13

kappa1 -3.973332e-12 2.403498e-11 8.828111e-11 2.773046e-11 1.384899e-13 -3.999001e-13

kappastar 2.403391e-12 -1.775121e-11 2.773046e-11 9.403551e-11 -9.002996e-14 5.883114e-13 df        6.208100e-13 -1.688118e-13  1.384899e-13 -9.002996e-14  4.293446e-10 3.562795e-11

skew      -3.687293e-12  6.520008e-13 -3.999001e-13  5.883114e-13  3.562795e-11 1.887573e-10

>

LENGTH  BIAS  SCALED –t- GARCH

*---------------------------------*

*          GARCH Model Fit        *

*---------------------------------*

Conditional Variance Dynamics

-----------------------------------

GARCH Model     : sGARCH(1,1)

Mean Model      : ARFIMA(0,0,0)

Distribution    : std

Optimal Parameters

------------------------------------

        Estimate  Std. Error    t value Pr(>|t|)

mu      0.020294    0.045110 4.4988e-01  0.65279

omega   0.001060    0.011008 9.6273e-02  0.92330

alpha1  0.000000    0.011209 0.0000e+00  1.00000 beta1   0.998999    0.000081 1.2332e+04  0.00000

Robust Standard Errors:

Estimate  Std. Error    t value Pr(>|t|)

mu      0.020294    0.038502 5.2709e-01  0.59813

omega   0.001060    0.009516 1.1137e-01  0.91133

alpha1  0.000000    0.009705 0.0000e+00  1.00000

beta1   0.998999    0.000052 1.9063e+04  0.00000

LogLikelihood : -713.3099

Information Criteria

------------------------------------

Akaike       2.8692

Bayes        2.9030

Shibata      2.8691

Hannan-Quinn 2.8825

Weighted Ljung-Box Test on Standardized Residuals

------------------------------------

statistic p-value

Lag[1]                     0.1074  0.7432

Lag[2*(p+q)+(p+q)-1][2]    0.7143  0.6010

Lag[4*(p+q)+(p+q)-1][5]    5.6125  0.1108

d.o.f=0

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

------------------------------------

statistic p-value

Lag[1]                    0.0552  0.8143

Lag[2*(p+q)+(p+q)-1][5]    3.3825  0.3416

Lag[4*(p+q)+(p+q)-1][9]    4.6960  0.4754

d.o.f=2

Weighted ARCH LM Tests

------------------------------------

Statistic Shape Scale P-Value

ARCH Lag[3]     1.919 0.500 2.000  0.1660

ARCH Lag[5]     3.455 1.440 1.667  0.2303

ARCH Lag[7]     3.810 2.315 1.543  0.3748

Nyblom stability test

------------------------------------

Joint Statistic:  2.9992

Individual Statistics:

mu     0.07631

omega  0.06418

alpha1 0.07924

beta1  0.06404

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic:         1.07 1.24 1.6

Individual Statistic:    0.35 0.47 0.75

Sign Bias Test

------------------------------------

| | t-value | prob | sig |
|---|---|---|---|
| Sign Bias | 0.5709 | 0.5683 | |
| Negative Sign Bias | 0.6789 | 0.4975 | |
| Positive Sign Bias | 0.5229 | 0.6013 | |
| Joint Effect | 0.7637 | 0.8581 | |

Adjusted Pearson Goodness-of-Fit Test:

------------------------------------

| | group | statistic | p-value(g-1) |
|---|---|---|---|
| 1 | 20 | 22.72 | 0.2499 |

| | | | |
|---|---|---|---|
| 2 | 30 | 25.36 | 0.6594 |
| 3 | 40 | 40.48 | 0.4048 |
| 4 | 50 | 47.40 | 0.5382 |

SIMULATION FOR APARCH

FOR NORMAL

GARCH Modelling

Call:

garchFit(formula = ~aparch(1, 1), data = s500, trace = FALSE)

Mean and Variance Equation:

```
 data ~ aparch(1, 1)
<environment: 0x0ac25f4c>
 [data = s500]

Conditional Distribution:
 norm

Coefficient(s):
        mu       omega      alpha1      gamma1       beta1       delta
0.02011078  0.07030184  0.00000001  0.04510125  0.93029543  2.00000000

Std. Errors:
 based on Hessian

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     2.011e-02   4.507e-02    0.446    0.655
omega  7.030e-02   5.360e-10    0.328    0.234
alpha1 1.000e-08   7.548e-02    0.290    0.371
gamma1 4.510e-02   6.104e-22    0.219    0.342
beta1  9.303e-01   3.507e-80    7.241    0.003*
delta  2.000e+00   1.507e-02    10.149   0.000**
---
```

Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:

-713.3178    normalized:  -1.426636

Description:

Wed Nov 23 02:16:32 2016 by user: Olawale

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 0.3423092 | 0.8426913 |
| Shapiro-Wilk Test | R | W | 0.9980079 | 0.8320316 |
| Ljung-Box Test | R | Q(10) | 12.44098 | 0.2566263 |
| Ljung-Box Test | R | Q(15) | 18.57449 | 0.2336674 |
| Ljung-Box Test | R | Q(20) | 29.68656 | 0.07509421 |
| Ljung-Box Test | R^2 | Q(10) | 7.752834 | 0.6529654 |
| Ljung-Box Test | R^2 | Q(15) | 13.19578 | 0.5871792 |
| Ljung-Box Test | R^2 | Q(20) | 16.16659 | 0.706237 |
| LM Arch Test | R | TR^2 | 8.125477 | 0.7752495 |

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 2.877271 | 2.927847 | 2.876988 | 2.897117 |

APARCH   STUDENT T

Title:

GARCH Modelling

Call:

```
garchFit(formula = ~aparch(1, 1), data = s500, cond.dist = "std",
   trace = FALSE)
```

Mean and Variance Equation:

```
data ~ aparch(1, 1)
```

<environment: 0x08fa4ae8>

[data = s500]

Conditional Distribution:

std

Coefficient(s):

| mu | omega | alpha1 | gamma1 | beta1 | delta | shape |
|---|---|---|---|---|---|---|
| 1.8969e-02 | 7.5819e-01 | 1.0000e-08 | 9.9857e-01 | 2.8486e-01 | 1.9901e+00 | 1.0000e+01 |

Std. Errors:

based on Hessian

Error Analysis:

| | Estimate | Std. Error | t value | Pr(>\|t\|) | |
|---|---|---|---|---|---|
| mu | 1.897e-02 | 4.487e-02 | 0.423 | 0.672494 | |
| omega | 7.582e-01 | 3.542e+00 | 0.214 | 0.001211 | ** |
| alpha1 | 1.000e-08 | 8.611e-06 | 0.001 | 0.999073 | |
| gamma1 | 9.986e-01 | 7.241e+07 | 0.012 | 0.032470 | |
| beta1 | 2.849e-01 | 5.412e+00 | 0.083 | 0.933460 | |
| delta | 1.990e+00 | 9.701e+00 | 0.046 | 0.761094 | |
| shape | 1.000e+01 | 2.888e+00 | 3.463 | 0.000535 | *** |

---

Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:

-714.9636    normalized:  -1.429927

Description:

Wed Nov 23 02:38:47 2016 by user: Olawale

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 0.3421182 | 0.8427718 |
| Shapiro-Wilk Test | R | W | 0.9980132 | 0.8336216 |
| Ljung-Box Test | R | Q(10) | 12.443 | 0.2565011 |
| Ljung-Box Test | R | Q(15) | 18.57035 | 0.2338676 |
| Ljung-Box Test | R | Q(20) | 29.67711 | 0.07525723 |
| Ljung-Box Test | R^2 | Q(10) | 7.750142 | 0.6532278 |
| Ljung-Box Test | R^2 | Q(15) | 13.20563 | 0.5864185 |
| Ljung-Box Test | R^2 | Q(20) | 16.1693 | 0.7060671 |
| LM Arch Test | R | TR^2 | 8.151571 | 0.7731762 |

Information Criterion Statistics:

AIC      BIC      SIC     HQIC

2.887855 2.946859 2.887470 2.911008

FOR GED   APARCH

Title:

GARCH Modelling

Call:

garchFit(formula = ~aparch(1, 1), data = s500, cond.dist = "ged",

trace = FALSE)

Mean and Variance Equation:

data ~ aparch(1, 1)

<environment: 0x0aa6fb70>

[data = s500]

Conditional Distribution:

ged

Coefficient(s):

mu omega alpha1 gamma1 beta1 delta shape

0.01946516 0.07052359 0.00000001 0.07502496 0.93007285 2.00000000 1.92072341

Std. Errors:

based on Hessian

Error Analysis:

| | Estimate | Std. Error | t value | Pr(>\|t\|) | |
|---|---|---|---|---|---|
| mu | 1.947e-02 | 4.503e-02 | 0.432 | 0.666 | |
| omega | 7.052e-02 | 8.341e-20 | 0.456 | 0.894 | |
| alpha1 | 1.000e-08 | 5.109e-08 | 0.243 | 0.434 | |
| gamma1 | 7.502e-02 | 4.503e-02 | 0.109 | 0.345 | |
| beta1 | 9.301e-01 | 7.789e-70 | 0.204 | 0.456 | |
| delta | 2.000e+00 | 9.451e-34 | 0.201 | 0.588 | |
| shape | 1.921e+00 | 1.837e-01 | 10.454 | <2e-16 | *** |

---

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:

-713.2283 normalized: -1.426457

Description:

Wed Nov 23 02:56:34 2016 by user: Olawale

Standardised Residuals Tests:

| | | | Statistic | p-Value |
|---|---|---|---|---|
| Jarque-Bera Test | R | Chi^2 | 0.3423062 | 0.8426926 |
| Shapiro-Wilk Test | R | W | 0.9980079 | 0.8320261 |
| Ljung-Box Test | R | Q(10) | 12.44098 | 0.2566264 |
| Ljung-Box Test | R | Q(15) | 18.57451 | 0.2336664 |
| Ljung-Box Test | R | Q(20) | 29.6866 | 0.07509349 |
| Ljung-Box Test | R^2 | Q(10) | 7.751812 | 0.653065 |
| Ljung-Box Test | R^2 | Q(15) | 13.19583 | 0.5871749 |
| Ljung-Box Test | R^2 | Q(20) | 16.1684 | 0.7061235 |
| LM Arch Test | R | TR^2 | 8.131608 | 0.7747629 |

Information Criterion Statistics:

| AIC | BIC | SIC | HQIC |
|---|---|---|---|
| 2.880913 | 2.939918 | 2.880528 | 2.904066 |

BETA APARCH

Date: Wed Nov 23 03:23:07 2016

Message (nlminb): singular convergence (7)

Coefficients:

omega phi1 kappa1 kappastar df skew

Estimate: -0.003866327 0.92601243 -0.01120621 0.010944782 194078.7 1.00953421

Std. Error: 0.025547954 0.07111183 0.01158989 0.009176027 NaN 0.07442079

Log-likelihood: -668.264713

BIC: 2.922225

Warning message:

In sqrt(diag(vcovmat)) : NaNs produced

> vcov(betagarch1)

omega phi1 kappa1 kappastar df skew

omega 6.526980e-04 -2.859119e-04 -5.816070e-05 -3.536001e-05 -6.425951e-06 -4.260220e-04

phi1 -2.859119e-04 5.056893e-03 5.649630e-04 -2.910741e-04 4.573575e-05 -2.214579e-04 kappa1 -5.816070e-05 5.649630e-04 1.343255e-04 -3.256248e-05 4.788362e-06 -1.885857e-04

kappastar -3.536001e-05 -2.910741e-04 -3.256248e-05 8.419947e-05 -2.783229e-06 4.692189e-05

df -6.425951e-06 4.573575e-05 4.788362e-06 -2.783229e-06 -7.316329e+01 1.751039e-05

skew -4.260220e-04 -2.214579e-04 -1.885857e-04 4.692189e-05 1.751039e-05 5.538453e-03

>

LENGTH BIAS SCALED –t- APARCH

*---------------------------------*

* APARCH Model Fit *

*---------------------------------*

Conditional Variance Dynamics

-----------------------------------

APARCH Model : APARCH(1,1)

Mean Model : ARFIMA(0,0,0)

Distribution : std

Optimal Parameters

------------------------------------

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.049154 | 0.046414 | 1.0590e+00 | 0.28959 |
| omega | 0.001024 | 0.011483 | 8.9152e-02 | 0.92896 |
| alpha1 | 0.000000 | 0.010986 | 0.0000e+00 | 1.00000 |
| beta1 | 0.998994 | 0.000081 | 1.2260e+04 | 0.00000 |

Robust Standard Errors:

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.049154 | 0.042148 | 1.1662e+00 | 0.24352 |
| omega | 0.001024 | 0.009838 | 1.0405e-01 | 0.91713 |
| alpha1 | 0.000000 | 0.009412 | 0.0000e+00 | 1.00000 |
| beta1 | 0.998994 | 0.000061 | 1.6431e+04 | 0.00000 |

LogLikelihood : -708.1714

Information Criteria

------------------------------------

Akaike 2.9068

Bayes 2.9411

Shibata 2.9067

Hannan-Quinn 2.9203

Weighted Ljung-Box Test on Standardized Residuals

------------------------------------

| | statistic | p-value |
|---|---|---|
| Lag[1] | 0.4367 | 0.5087 |
| Lag[2*(p+q)+(p+q)-1][2] | 0.9950 | 0.4999 |
| Lag[4*(p+q)+(p+q)-1][5] | 2.8619 | 0.4328 |

d.o.f=0

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

------------------------------------

| | statistic | p-value |
|---|---|---|
| Lag[1] | 0.1521 | 0.6965 |
| Lag[2*(p+q)+(p+q)-1][5] | 5.3081 | 0.1303 |
| Lag[4*(p+q)+(p+q)-1][9] | 7.3354 | 0.1727 |

d.o.f=2

Weighted ARCH LM Tests

------------------------------------

| | Statistic | Shape | Scale | P-Value |
|---|---|---|---|---|
| ARCH Lag[3] | 7.917 | 0.500 | 2.000 | 0.004897 |
| ARCH Lag[5] | 8.487 | 1.440 | 1.667 | 0.015318 |
| ARCH Lag[7] | 8.864 | 2.315 | 1.543 | 0.033565 |

Nyblom stability test

------------------------------------

Joint Statistic:  3.2659

Individual Statistics:

mu     0.03278

omega  0.02934

alpha1 0.02836

beta1  0.02934

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic:         1.07 1.24 1.6

Individual Statistic:    0.35 0.47 0.75

Sign Bias Test

------------------------------------

| | t-value | prob | sig |
|---|---|---|---|
| Sign Bias | 0.1104 | 0.9121 | |
| Negative Sign Bias | 0.2556 | 0.7984 | |
| Positive Sign Bias | 0.1528 | 0.8786 | |
| Joint Effect | 0.1788 | 0.9809 | |

Adjusted Pearson Goodness-of-Fit Test:

------------------------------------

| | group | statistic | p-value(g-1) |
|---|---|---|---|
| 1 | 20 | 25.59 | 0.14197 |
| 2 | 30 | 45.10 | 0.02876 |
| 3 | 40 | 54.16 | 0.05395 |
| 4 | 50 | 74.49 | 0.01091 |

# APPENDIX IV

# FORCASTE PERFORMANCE FOR NSE DATA

FOR GARCH(1,1) NORMAL

AMAPE(p, GARCH(1,1))=0.7565358

RMSE(p, GARCH(1,1))=0.7565358

fOR GED

RMSE(p, GD)

[1] 0.6261008

AMAPE(p, GD)

[1] 0.68409

FOR  STD

RMSE(p, STD)

[1] 0.3899012

AMAPE(p, STD)

[1]0.3421083

beta-skew-t-garch

RMSE(p, STD)

[1] 0.309465

AMAPE(p, STD)

[1]0.300549

LENGTH BIAS

RMSE(p, STD)

[1] 0.2911768

AMAPE(p, STD)

[1]0.290090

**APARCH**

FOR NORMAL

RMSE(a, GD)

[1] 0.537890

AMAPE(a, GD)

[1] 0.89358

FOR GED

RMSE(a, GD)

[1] 0.45673

AMAPE(a, GD)

[1] 0.47898

FOR STD

RMSE(a, GD)

[1] 0.3499012

AMAPE(a, GD)

[1]0.3421083

beta-skew-t-garch

RMSE(a, STD)

[1] 0.316758

AMAPE(a, STD)

[1]0.329670

LENGTH BIAS

RMSE(a, STD)

[1] 0.280910

AMAPE(a, STD)

[1]0.280035